1. **38 points** Consider a set of 10 graduating seniors. Assume that their salaries \( X \) (in increments of $10,000) are \( \{4, 6, 6, 7, 7, 9, 9, 10, 11\} \).

Let \( F_X \) be the cumulative distribution function of \( X \).

(a) **8 points** Compute \( F_X(8) \).

(b) **8 points** Compute \( \mathbb{E}[X] \).

(c) **8 points** Compute \( \mathbb{E}[X^2] \).

(d) **6 points** Compute the variance of \( X \).

(e) **8 points** Compute \( \mathbb{E}[7X + 9] \).

2. **22 points** Suppose that \( \mathbb{P}(A) = 0.2 \quad \mathbb{P}(B) = 0.8 \quad \text{and} \quad \mathbb{P}(A|B) = 0.4 \).

(a) **8 points** Compute \( \mathbb{P}(A \cap B) \)

(b) **8 points** Compute \( \mathbb{P}(A \cup B) \)

(c) **6 points** Compute \( \mathbb{P}(B \setminus A) \) (recall that \( B \setminus A \overset{\text{def}}{=} B \cap A^c \)).

3. **22 points** Professor X gives her students a multiple choice exam. Each question has 5 possible answers. Sam spent the week preparing for the exam. He studied 70% of the chapters, read 20% of the chapters, and disregarded the remaining 10% of the chapters. If he studied a given chapter, he knows the answers to questions posed from that chapter. If he read the chapter, he will correctly answer a question from that chapter with probability .75 (i.e., 75%). If he disregarded the chapter, he guesses.

(a) **8 points** If Sam gets the correct answer, what is the probability that he has studied the chapter?

(b) **8 points** If Sam gets the correct answer, what is the probability that he has read the chapter?

(c) **6 points** If Sam gets the correct answer, what is the probability that he has studied or read the chapter?
4. 18 points Suppose that $X$ has cumulative distribution function

$$F_X(t) = \begin{cases} 
0 & \text{if } t < 0 \\
\frac{2}{3}t & \text{if } 0 \leq t < 1 \\
1 & \text{if } t \geq 1 
\end{cases}$$

(a)  9 points Compute $\mathbb{P}\{X > \frac{1}{2}\}$.

(b)  9 points Compute $\mathbb{P}\{X = 1\}$. 
Answers

1. (a) 

\[ F_X(8) = \frac{6}{10}. \]

(b) 

\[ E[X] = \frac{1}{10} \{4 + 6 + 6 + 7 + 7 + 7 + 9 + 9 + 10 + 11\} = 7.6. \]

(c) 

\[ E[X^2] = \frac{1}{10} \{4^2 + 6^2 + 6^2 + 7^2 + 7^2 + 7^2 + 9^2 + 9^2 + 10^2 + 11^2\} \]
\[ = \frac{1}{10} \{16 + 36 + 36 + 49 + 49 + 49 + 81 + 81 + 100 + 121\} = \frac{569}{10} = 61.8. \]

(d) 

\[ E[X^2] - E[X]^2 = 61.8 - 56.25 = 5.55 \]

(e) 

\[ E[7X + 9] = 7E[X] + 9 = 7 \times 7.6 + 9 = 62.2. \]

2. (a) 

\[ P(A \cap B) = P(A|B)P(B) = 0.4 \times 0.8 = .32. \]

(b) 

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.8 - .32 = .68. \]

(c) 

\[ P(B \setminus A) = P(B) - P(B \cap A) = 0.8 - .32 = .48. \]

3. Define 

\[ S \overset{\text{def}}{=} \{\text{studied}\}, \quad R \overset{\text{def}}{=} \{\text{read}\}, \quad \text{and} \quad D \overset{\text{def}}{=} \{\text{disregarded}\}. \]

Then 

\[ P(S) = .7 \quad P(R) = .20, \quad \text{and} \quad P(D) = .1. \]

Defining \( C \overset{\text{def}}{=} \{\text{correct answers}\}, \) we have that 

\[ P(C|S) = 1 \quad P(C|R) = .75 \quad \text{and} \quad P(C|D) = .2. \]

(a) 

\[ P(S|C) = \frac{P(S \cap C)}{P(C)} = \frac{P(C \cap S)}{P(C \cap S) + P(C \cap R) + P(C \cap D)} \]
\[ = \frac{P(C|S)P(S)}{1 \times .7 + .75 \times .2 + .2 \times .1} = \frac{.7}{.87}. \]
(b) 

\[ P(R|C) = \frac{P(R \cap C)}{P(C)} = \frac{P(C \cap R)}{P(C | R)P(R) + P(C | S)P(S) + P(C | D)P(D)} \]

\[ = \frac{.75 \times .2}{1 \times .7 + .75 \times .2 + .2 \times .1} = \frac{.15}{.87}. \]

(c) 

\[ P(S \cup R|C) = P(S|C) + P(R|C) = \frac{.7}{.87} + \frac{.15}{.87} = \frac{85}{.87}. \]

4. (a) 

\[ P\{X > \frac{1}{2}\} = 1 - P\{X \leq \frac{1}{2}\} = 1 - F_X(1/2) = 1 - \frac{1}{3} = \frac{2}{3}. \]

(b) 

\[ P\{X = 1\} = F_X(t) - F_X(1-) = 1 - \frac{2}{3} = \frac{1}{3}. \]