Show all work; partial credit will be given
No calculators allowed
Remember that I have to grade this; don’t over-reduce your answers
Remember that I have to grade this; be neat

1. **10 points** (Similar to Question 43 in Chapter 3). There are 4 coins in a box:
   - A two-headed coin
   - Two fair coins
   - A biased coin that comes up heads 75% of the time.

   One of the coins is randomly selected and tossed. If it shows up heads, what is the probability that it is the two-headed coin?

2. **30 points** Suppose that
   \[ P(A \cap C|B) = .2, \quad P(A|B) = .6, \quad \text{and} \quad P(A \cap B) = .4. \]

   Let’s do some calculations. Do not confuse \( \setminus \), which denotes set subtraction, with \( | \), which denotes conditional probability.
   
   (a) **10 points** Compute \( P(B) \).
   
   (b) **10 points** Compute \( P(A \setminus C|B) \).
   
   (c) **10 points** Compute \( P(A \cap B \cap C) \)

3. **30 points** Suppose that Alex and Bob are on a team of 20 people. Two committees will be formed; committee \( A \) contains 8 people and committee \( B \) contains 10 people. The committees can overlap.
   
   (a) **10 points** How many ways can the committees be formed?
   
   (b) **10 points** What is the probability that Alex and Bob will both be on committee \( A \).
   
   (c) **10 points** What is the probability that Alex and Bob will both be on committee \( A \) but they will not both be on committee \( B \)?

4. **30 points** 4 couples are in a room and to be seated at a dinner table. Each side of the table can accommodate 4 people.
   
   (a) **10 points** How many ways can the people be seated if all the men are on one side, all the women are on the other side?
   
   (b) **10 points** How many ways can the people be seated if each husband and wife are to be seated across from each other?
   
   (c) **10 points** How many ways can the people be seated if each husband and wife are to be seated across from each other, and each person’s neighbor is of the opposite gender?
1. 
\[ P(C_1 | H) = \frac{P(C_1 \cap H)}{P(H)} = \frac{P(H | C_1)P(C_1)}{P(H | C_1)P(C_1) + P(H | C_2)P(C_2) + P(H | C_3)P(C_3) + P(H | C_4)P(C_4)} = 1 \cdot \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{4}}. \]

2. (a) 
\[ P(B) = \frac{P(A \cap B)}{P(A | B)} = \frac{.4}{.5} = \frac{2}{3}. \]

(b) \( P(A \setminus C | B) = P(A | B) - P(A \cap C | B) = .6 - .2 = .4 \)

(c) \( P(A \cap B \cap C) = P(A \cap C | B)P(B) = .2 \times \frac{2}{3} \).

3. (a) \((\binom{20}{8})\binom{20}{10}\).

(b) 
\[ \frac{(\binom{18}{6})\binom{20}{10}}{(\binom{20}{8})(\binom{20}{10})} = \frac{(\binom{18}{6})}{(\binom{20}{8})}. \]

(c) 
\[ P\{\text{Alex and Bob are both on committee A}\} - P\{\text{Alex and Bob are both on committees A and B}\} = \frac{(\binom{18}{6})}{(\binom{20}{8})} - \frac{(\binom{18}{6})(\binom{18}{8})}{(\binom{20}{8})(\binom{20}{10})}. \]

4. (a) \(4! \times 4!\)

(b) \(2^4 \times 4!\)

(c) \(2 \times 4!\)