1. **20 points** (Question 37 in Chapter 2) An instructor gives her class a set of 10 study problems to figure out in preparation for an exam. The exam will consist of 5 of these problems and 5 other problems. Sammy Student has figured out 7 of the study problems. Also, he knows about 80% of the material in the chapter; i.e., for a typical random question from the material, he will get it right 80% of the time.

   (a) **10 points** What is the probability that he will get a perfect score on the exam?

   (b) **10 points** What is the probability that he will 90% on the exam?

2. **40 points** Suppose that we want to arrange our textbooks on a shelf. Suppose that we have

   - Language textbooks in Spanish, Mandarin Chinese, and Urdu (3 total)
   - Novels (i.e., literature textbooks) by José Saramago, Djuna Barnes, Norman Mailer, and Maya Angelou (4 total)
   - Mathematics textbooks in Probability, Calculus, Differential Equations, Statistics, and Number Theory (5 total)
   - Science textbooks in Physics, Biology, Cosmology, Geology, Organic Chemistry, and Inorganic Chemistry (6 total)

   Suppose that we want to keep the language textbooks together, the literature textbooks together, the mathematics textbooks together, and the science textbooks together.

   (a) **10 points** How many ways are there to arrange the books?

   (b) **10 points** What is the probability that José Saramago will be put next to the calculus book?

   Suppose now that we need to sell 2 books

   (c) **10 points** How many ways can we do this if both books are from the same field (i.e., languages, literature, math, or science)?

   (d) **10 points** How many ways can we do this if both books are from different fields (i.e., languages, literature, math, or science)?

3. **20 points** Players $A$ and $B$ alternate rolling a pair of dice ($A$ starts) stopping when $A$ wins by rolling a 7 or $B$ wins by rolling a 5.

   (a) **10 points** Compute the probability that $B$ wins on her 3-rd toss.

   (b) **10 points** Compute the probability that $B$ wins.
4. 20 points (Like example 3m). Suppose that a box contains 100 flashlights. Twenty of the flashlights are type A, 30 are of type B and the remaining 50 are of type C. Flashlights of type A work 90% of the time, flashlights of type B work 70% of the time, and flashlights of type C work 60% of the time.

(a) 10 points What is the probability that a randomly selected flashlight will work?

(b) 10 points If a randomly selected flashlight works, what is the probability that it is either type A or B?
1. (a) \((0.8)^5 \frac{3}{10^5}\)

(b) \((0.8)^4(0.2)\frac{3}{10^5} + (0.8)^5 \frac{3}{10^5}\).

2. (a) \(3! \times 4! \times 5! \times 6! \times 4!\)

(b) \(3! \times 4! \times 3! \times 6! \times 2 \times 3! = 3! \times 4! \times 2 \times 3! = \frac{2}{4 \times 5 \times 4} = \frac{1}{40}\)

Explaination of numerator: 3! ways to order non-Saramago lit books, 4! ways to order non-calculus math books. 3! ways to order language books. 6! ways to order science books. 2 for lit then math as opposed to math then lit, then 3! ways to order subjects, where lit-math is grouped together.

(c) \(\left(\binom{3}{2}\right) + \left(\binom{4}{2}\right) + \left(\binom{5}{2}\right) + \left(\binom{6}{2}\right)\).

(d) \(3 \times 4 + 3 \times 5 + 3 \times 6 + 4 \times 5 + 4 \times 6 + 5 \times 6\).

3. Note that for a single toss of the dice

\(\mathbb{P}\{7\} = \frac{6}{36}\) and \(\mathbb{P}\{5\} = \frac{4}{36}\).

(a) Call the set in question \(B_3\). Then

\(\mathbb{P}(B_3) = \left(\frac{32}{36}\right)^2 \frac{4}{36} \left(\frac{30}{36}\right)^3\).

(b)

\(\mathbb{P}\{B \text{ wins}\} = \sum_{n=1}^{\infty} \left(\frac{32}{36}\right)^{n-1} \frac{4}{36} \left(\frac{30}{36}\right)^n = \frac{4}{36} \frac{30}{36} \sum_{n=1}^{\infty} \left(\frac{32}{36}\right)^{n-1}\)

\(= \frac{4}{36} \frac{30}{36} \sum_{j=0}^{\infty} \left(\frac{32}{36}\right)^j = \frac{(4/36)(30/36)}{1 - (32/36)(30/36)} = \frac{4 \times 30}{36^2 - 32 \times 30}.

4. (a)

\(\mathbb{P}(W) = \mathbb{P}(W|A)\mathbb{P}(A) + \mathbb{P}(W|B)\mathbb{P}(B) + \mathbb{P}(W|C)\mathbb{P}(C) = 0.9 \times 0.2 + 0.7 \times 0.3 + 0.6 \times 0.5\).

R. Sowers 3
(b) 

\[ P(A \cup B | W) = \frac{P((A \cup B) \cap W)}{P(W)} = \frac{P(W | A)P(A) + P(W | B)P(B)}{P(W | A)P(A) + P(W | B)P(B) + P(W | C)P(C)} \]

\[ = \frac{0.9 \times 0.2 + 0.7 \times 0.3}{0.9 \times 0.2 + 0.7 \times 0.3 + 0.6 \times 0.5}. \]