1. **10 points** Let $U$ be a Uniform(0, 1) continuous random variable; i.e., it has density

$$f_U(t) \overset{\text{def}}{=} \begin{cases} 1 & \text{if } 0 < t < 1 \\ 0 & \text{else} \end{cases}$$

Define the transformation

$$g(u) \overset{\text{def}}{=} \begin{cases} 1 - u & \text{if } u \leq \frac{2}{3} \\ u & \text{if } u > \frac{2}{3} \end{cases}$$

and define $X \overset{\text{def}}{=} g(U)$.

(a) **5 points** Compute $F_X$, the cumulative distribution function of $X$.

(b) **5 points** Compute $f_X$, the density of $X$. 
1. (a) \( F_X(t) = 1 \) if \( t \geq 1 \) and \( F_X(t) = 0 \) for \( t < 0 \). For \( \frac{1}{3} \leq t < \frac{2}{3} \), we have that
\[
F_X(t) = P\left\{ 1 - t \leq U \leq \frac{2}{3} \right\} = \frac{2}{3} - (1 - t) = t - \frac{1}{3}.
\]
For \( \frac{2}{3} \leq t < 1 \), we have that
\[
F_X(t) = P\{1 - t \leq U \leq t\} = t - (1 - t) = 2t - 1.
\]
Thus
\[
F_X(t) = \begin{cases} 
0 & \text{if } t < 0 \\
\frac{t - 1}{3} & \text{if } \frac{1}{3} \leq t < \frac{2}{3} \\
2t - 1 & \text{if } \frac{2}{3} \leq t < 1 \\
1 & \text{if } t \geq 1 
\end{cases}
\]
(b) Differentiate \( F_X \) to get that
\[
f_X(t) = \begin{cases} 
1 & \text{if } \frac{1}{3} < t < \frac{2}{3} \\
2 & \text{if } \frac{2}{3} < t < 1 \\
0 & \text{else}
\end{cases}
\]