

1. 10 points Let X be a random variable with moment generating function

$$M(\theta) \stackrel{\text{def}}{=} \mathbb{E}[e^{\theta X}] = \exp\left[\frac{9}{2}\theta^2\right]. \quad \theta \in \mathbb{R}$$

Define $Y \stackrel{\text{def}}{=} 2X + 7$. Compute the moment generating function of Y .

ANSWERS

1.

$$\begin{aligned}\mathbb{E}[\exp[\theta Y]] &= \mathbb{E}[\exp[\theta(2X + 7)]] = \mathbb{E}[\exp[(2\theta)X + 7\theta]] \\ &= \mathbb{E}[\exp[(2\theta)X] e^{7\theta}] = e^{7\theta} \mathbb{E}[\exp[(2\theta)X]] = e^{7\theta} \exp\left[\frac{9}{2}(2\theta)^2\right] \\ &= \exp[18\theta^2 + 7\theta].\end{aligned}$$