

1. Let X be a continuous random variable with density

$$f(x) = \begin{cases} Cx^3 & \text{if } 0 \leq x \leq 2 \\ 0 & \text{else} \end{cases}$$

(a) 5 points Find the constant C .

(b) 5 points Compute $\mathbb{E}[X^2]$.

ANSWERS

1. (a) We must have

$$1 = \int_{x=-\infty}^{\infty} f(x)dx = C \int_{x=0}^2 x^3 dx = C \frac{2^4}{4} = 4C$$

so $C = 1/4$.

(b) We compute that

$$\mathbb{E}[X^2] = \int_{x=-\infty}^{\infty} x^2 f(x) dx = \frac{1}{4} \int_{x=0}^2 x^2 x^3 dx = \frac{1}{4} \frac{2^5}{5} = \frac{32}{4 \times 5} = \frac{8}{5}.$$