Math 361, Section D13 and D14, Fall 2007
Quiz 6, October 8 **Name:** ______________________

1. 10 points This is a review of useful calculus. Prove that for any real number λ,

$$\lim_{n\to\infty}\left(1+\frac{\lambda}{n}\right)^n = e^{\lambda}.$$

If you use another result from calculus, clearly state what you are using.

Answers

1. We first note that by L'Hôpiltal's rule,

$$\lim_{x\nearrow\infty} x\ln\left(1+\frac{\lambda}{x}\right) = \lim_{x\nearrow\infty}\frac{\ln\left(1+\frac{\lambda}{x}\right)}{\frac{1}{x}} = \lim_{x\searrow 0}\frac{\ln(1+\lambda x)}{x} = \lim_{x\searrow 0}\frac{\lambda}{\ln(1+\lambda x)} = \lambda.$$

Since the exponential map is continuous,

$$\lim_{n\to\infty}\left(1+\frac{\lambda}{n}\right)^n = \exp\left[\lim_{x\nearrow\infty} x\ln\left(1+\frac{\lambda}{x}\right)\right] = e^{\lambda}.$$