1. **10 points** Consider an unfair die with

\[
P\{1\} = P\{2\} = P\{3\} = P\{4\} = \frac{1}{6} \quad P\{5\} = \frac{1}{12} \quad \text{and} \quad P\{6\} = \frac{3}{12}.
\]

Assume also that \(X\) is a random variable defined as

\[X(1) = X(2) = 1, \quad X(3) = X(4) = 2, \quad \text{and} \quad X(5) = X(6) = 3.\]

(a) **5 points** Compute \(E[X]\).

(b) **5 points** Compute \(E[X^2]\).

You don’t need to reduce the answers.
Answers

1. (a)

\[ E[X] = \sum_{j=1}^{6} X(j) \mathbb{P}\{j\} = 1 \times \frac{1}{6} + 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{12} + 3 \times \frac{3}{12}. \]

(b)

\[ E[X^2] = \sum_{j=1}^{6} X^2(j) \mathbb{P}\{j\} = 1 \times \frac{1}{6} + 1 \times \frac{1}{6} + 4 \times \frac{1}{6} + 4 \times \frac{1}{6} + 9 \times \frac{1}{12} + 9 \times \frac{3}{12}. \]