1. 10 points This is question 62 on page 119. Barbara and Dianne shoot at a target. They are independent shooters. Suppose that Barbara hits the target with probability $p_B$ and Dianne hits the target with probability $p_D$. Suppose that we know that the target was hit.

(a) 5 points What is the probability that Barbara hit the target?

(b) 5 points What is the probability that both Barbara and Dianne hit the target?
1. Let 

\[ B = \{ \text{Barbara hits the target} \} \quad D = \{ \text{Dianne hits the target} \}. \]

The target being hit is the set \( B \cup D \), and

\[ \mathbb{P}(B \cup D) = \mathbb{P}(B) + \mathbb{P}(D) - \mathbb{P}(B \cap D) = p_B + p_D - p_B p_D. \]

(a) Note that \( B \subset (B \cup D) \), so

\[ \mathbb{P}(B \mid B \cup D) = \frac{\mathbb{P}(B)}{\mathbb{P}(B \cup D)} = \frac{p_B}{p_B + p_D - p_B p_D}. \]

(b) Note that \( B \cap D \subset (B \cup D) \), so

\[ \mathbb{P}(B \cap D \mid B \cup D) = \frac{\mathbb{P}(B \cap D)}{\mathbb{P}(B \cup D)} = \frac{p_B p_D}{p_B + p_D - p_B p_D}. \]