**Math 361, Section D13 and D14, Fall 2007**
**Quiz 11, December 3** **Name:** ____________________

1. Let $X$ and $Y$ be independent continuous random variables. Suppose that $X$ is uniform $(0,10)$ and $Y$ is uniform $(0,5)$. In other words,

$$f_X(t) = \begin{cases} \frac{1}{10} & \text{if } 0 < t < 10 \\ 0 & \text{else} \end{cases}$$

$$f_Y(t) = \begin{cases} \frac{1}{5} & \text{if } 0 < t < 5 \\ 0 & \text{else} \end{cases}$$

Define $Z \stackrel{\text{def}}{=} X + Y$, and let $f_Z$ be the density of $Z$.

(a) [3 points] Compute $\mathbb{E}[X]$.

(b) [7 points] Compute $f_Z$.

## Answers

1. (a) $\mathbb{E}[X] = \int_{s=-\infty}^{\infty} s f_X(s) ds = \frac{1}{10}\int_{s=0}^{10} s ds = 5.$

   (b) We can write

$$f_Z(t) = \frac{1}{10}\chi_{(0,10)}(t) \qquad \text{and} \qquad f_Y(t) = \frac{1}{5}\chi_{(0,5)}(t)$$

for all $t \in \mathbb{R}$. Then

$$\begin{aligned}
f_Z(t) &= \int_{s=-\infty}^{\infty} f_X(s) f_Y(t-s) ds = \frac{1}{50}\int_{s=-\infty}^{\infty} \chi_{(0,10)}(s)\chi_{(0,5)}(t-s) ds \\
&= \frac{1}{50}\int_{s=-\infty}^{\infty} \chi_{(0,10)}(s)\chi_{(t-5,t)}(s) ds = \begin{cases} \frac{1}{50}\int_{s=0}^{t} ds & \text{if } t-5<0<t \\ \frac{1}{50}\int_{s=t-5}^{t} ds & \text{if } 0<t-5<10 \\ \frac{1}{50}\int_{s=t-5}^{10} ds & \text{if } t-5<10<t \\ 0 & \text{else} \end{cases} \\
&= \begin{cases} \frac{t}{50}\int_{s=0}^{t} ds & \text{if } 0<t<5 \\ \frac{5}{50}\int_{s=t-5}^{t} ds & \text{if } 5<t<10 \\ \frac{10-(t-5)}{50} & \text{if } 10<t<15 \\ 0 & \text{else} \end{cases} = \begin{cases} \frac{t}{50} & \text{if } 0<t<5 \\ \frac{5}{50} & \text{if } 5<t<10 \\ \frac{15-t}{50} & \text{if } 10<t<15 \\ 0 & \text{else} \end{cases}
\end{aligned}$$