

1. Let X and Y continuous random variables with joint density function

$$f_{X,Y}(s, t) = \begin{cases} e^{-t} & \text{if } 0 \leq s \leq t \\ 0 & \text{else} \end{cases}$$

- (a) Graph the area where $f_{X,Y} \neq 0$.
- (b) Find f_X
- (c) Find f_Y

ANSWERS

1. (a) $f_{X,Y}(s,t)$ is nonzero above the line $s = t$ and in the first quadrant.

(b)

$$f_X(s) = \int_{t=-\infty}^{\infty} f_{X,Y}(s,t)dt = \begin{cases} \int_{t=s}^{\infty} e^{-t}dt & \text{if } s > 0 \\ 0 & \text{else} \end{cases} = \begin{cases} e^{-s} & \text{if } s > 0 \\ 0 & \text{else} \end{cases}$$

(c)

$$f_Y(t) = \int_{s=-\infty}^{\infty} f_{X,Y}(s,t)ds = \begin{cases} \int_{s=0}^t e^{-s}ds & \text{if } t > 0 \\ 0 & \text{else} \end{cases} = \begin{cases} te^{-t} & \text{if } t > 0 \\ 0 & \text{else} \end{cases}$$