Math 361, Section D13 and D14, Fall 2007
Exam 1, September 17 **Name:** _____

1. [50 points] This is question 52 on page 60. It is similar to the hat-matching problem.

   A box contains 10 pairs of shoes (we can label them as pairs 1 through 10). We grab 8 shoes and put them into a bag.

   (a) [10 points] What is the probability that the 1-st pair of shoes will be in the bag?

   (b) [10 points] What is the probability that the 4th and 7th pairs of shoes will be in the bag?

   (c) [10 points] What is the probability that the 2nd, 3rd, and 8th pairs of shoes will be in the bag?

   (d) [10 points] What is the probability that the 1st, 4th, 5th, and 10th pairs of shoes will be in the bag?

   (e) [10 points] What is the probability that there will be at least one (i.e., matching) pair of shoes in the bag? (Hint: use the inclusion-exlusion principle).

2. [15 points] This is essentially question 1 on page 111. Two fair dice are rolled.

   (a) [5 points] What is the probability that the two dice will show different numbers?

   (b) [10 points] What is the probability that at least one dice shows a 6 given that they show different numbers?

3. [15 points] This is essentially question 17 on page 112. In a certain community, $\frac{1}{3}$ of the people own a dog and $\frac{1}{5}$ own a cat. It also turns out that 10% of the dog-owners also own a cat.

   (a) [10 points] What is the probability that a randomly-selected family owns both a cat and a dog?

   (b) [5 points] What is the probability that a randomly-selected family which owns a dog given that it owns a cat?

4. [20 points] This question was motivated by problem 47 on page 60. There are 10 people in a room. Jane and Larry are among them.

   (a) [10 points] What is the probability that Jane and Larry were born in February and none of the other people in the room were born in February?

   (b) [10 points] What is the probability that exactly two of the people in the room were born in the same month?

Answers

1. (a)
$$\frac{\binom{2}{2}\binom{18}{6}}{\binom{20}{8}} = \frac{\binom{18}{6}}{\binom{20}{8}} = \frac{(8)_2}{(20)_2}.$$

(b)
$$\frac{\binom{4}{4}\binom{16}{4}}{\binom{20}{8}} = \frac{\binom{16}{4}}{\binom{20}{8}} = \frac{(8)_4}{(20)_4}.$$

(c)
$$\frac{\binom{6}{6}\binom{14}{2}}{\binom{20}{8}} = \frac{\binom{14}{2}}{\binom{20}{8}} = \frac{(8)_6}{(20)_6}.$$

(d)
$$\frac{\binom{8}{8}}{\binom{20}{8}} = \frac{1}{\binom{20}{8}} = \frac{8!}{(20)_6}.$$

(e)
$$\frac{10\binom{18}{6} - \binom{10}{2}\binom{16}{4} + \binom{10}{3}\binom{14}{2} - \binom{10}{4}}{\binom{20}{8}} = 1 - \frac{\binom{10}{8}2^8}{\binom{20}{8}}.$$

2. (a)
$$\frac{(6)_2}{36} = \frac{6 \times 5}{36}.$$

(b)

$$\mathbb{P}\{\text{at least one dice shows a 6 and they show different numbers}\}$$
$$= \mathbb{P}\{\text{one dice shows a 6 and they show different numbers}\} = \frac{2 \times 5}{36}.$$

Thus the answer to the question is
$$\frac{2 \times 5}{6 \times 5} = \frac{2}{5}.$$

3. We have that
$$\mathbb{P}(D) = \frac{1}{3}, \qquad \mathbb{P}(C) = \frac{1}{5}, \qquad \text{and} \qquad \mathbb{P}(C|D) = \frac{1}{10}.$$

(a)
$$\mathbb{P}(C \cap D) = \mathbb{P}(C|D)\mathbb{P}(D) = \frac{1}{10} \times \frac{1}{3} = \frac{1}{30}.$$

(b)
$$\mathbb{P}(D|C) = \frac{\mathbb{P}(D \cap C)}{\mathbb{P}(C)} = \frac{\frac{1}{30}}{\frac{1}{5}} = \frac{5}{30} = \frac{1}{6}.$$

4. (a)

$$\frac{11^8}{12^{10}}.$$

(b)

$$12\binom{10}{2}\frac{(11)_8}{12^{10}}.$$