1. **5 points** Find all solutions of the ODE

\[ \ddot{y} + 4y = 0. \]

2. **5 points** Find all solutions of the ODE

\[ \ddot{y} + 4y = 16t. \]
Answers

1. Characteristic equation is \( \lambda^2 + 4 = 0 \); roots are \( \lambda = \pm 2i \). All solutions are of the form

\[
y(t) = C_1 \cos(2t) + C_2 \sin(2t).
\]

2. Consider \( y_0(t) \overset{\text{def}}{=} t^n \); then

\[
\ddot{y}_0 + 4y_0 = n(n - 1)t^{n-2} + 4t^n.
\]

Want to take \( n = 1 \). Consider \( y(t) = a_1 t + a_0 \); then

\[
16t = \ddot{y}_0(t) + 4y_0(t) = 4(a_1 t + a_0);
\]

take \( a_1 = 4 \) and \( a_0 = 0 \). All solutions are

\[
y(t) = 4t + C_1 \cos(2t) + C_2 \sin(2t)
\]