1. **10 points** Suppose that $X$ is a standard Gaussian random variable, i.e., it is continuous with density

$$f_X(t) \overset{\text{def}}{=} \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{t^2}{2} \right] \quad t \in \mathbb{R}$$

Define $Y \overset{\text{def}}{=} -7X + 5$.

(a) **3 points** Compute $\mathbb{P}\{Y \leq 40\}$ (in terms of $f_X$).

(b) **2 points** Compute the cumulative distribution function $F_Y$ (in terms of $f_X$).

(c) **5 points** Compute the density $f_Y$ of $Y$ (the answer should be explicit).
Answers

1. (a) \[ P\{Y \leq 40\} = P\{Y \geq -5\} = \int_{t=-5}^{\infty} f_X(s)ds \]

(b) \[ F_Y(t) = P\{Y \leq t\} = P\left\{ Y \geq \frac{5-t}{7} \right\} = \int_{s=(5-t)/7}^{\infty} f_X(s)ds \]

(c) \[ f_Y(t) = \frac{1}{\sqrt{2\pi(49)}} \exp \left[ -\frac{(t-5)^2}{2(49)} \right] \quad t \in \mathbb{R} \]