1. [10 points] Let $X$ and $Y$ be independent random variables with distributions

$$f_X(j) = \begin{cases} (1 - p)^j & \text{if } j \in \{11, 12 \ldots \} \\ 1 - (1 - p)^{11} & \text{if } j = 10 \\ 0 & \text{else} \end{cases}$$

$$f_Y(j) = \begin{cases} (1 - p)^j & \text{if } j \in \{0, 1, 2 \ldots \} \\ 0 & \text{else} \end{cases}$$

Define $Z \overset{\text{def}}{=} X + Y$. Compute $f_Z(15)$. 
Answers

1.

\[ f_x(j) = 5p^2(1 - p)^{15} + p(1 - (1 - p)^1)(1 - p)^5. \]