1. **10 points** Suppose that we toss a fair six-sided die 3 times. Define

\[ A \overset{\text{def}}{=} \{ \text{face 2 appeared at least once} \} \]
\[ B \overset{\text{def}}{=} \{ \text{face 2 appeared exactly twice} \} \]

(a) **5 points** What is \( P(B|A) \)?

(b) **5 points** Are the sets \( A \) and \( B \) independent?
Answers

1. Since $B \subset A$, $\mathbb{P}(B \cap A) = \mathbb{P}(B)$, so

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(B)}{\mathbb{P}(A)}.$$ 

We can compute that

$$\mathbb{P}(B) = 3 \left( \frac{1}{6} \right)^2 \left( \frac{5}{6} \right)$$

$$\mathbb{P}(A) = 1 - \mathbb{P}\{\text{face 2 doesn’t appear at all}\} = 1 - \left( \frac{5}{6} \right)^3$$

(a) 

$$\mathbb{P}(B|A) = \frac{(3)(5)}{6^3 - 5^3}$$

(b) Since $\mathbb{P}(A) < 1$, $\mathbb{P}(B|A) > \mathbb{P}(B)$, so $\mathbb{P}(B|A) \neq \mathbb{P}(A)$. Thus they are dependent.