1. [10 points] Consider a certain test for cancer. Suppose that 90% of those with cancer react positively to the test, while 5% of those without cancer also react positively to the test. Suppose also that 1% of people actually have cancer. Suppose you take the test and react positively. What is the probability that you actually have cancer?
1. Define the sets

\[ P \overset{\text{def}}{=} \{ \text{test positive} \} \quad \text{and} \quad C = \{ \text{have cancer} \}. \]

Then

\[ \mathbb{P}(P|C) = 0.9, \quad \mathbb{P}(P|C^c) = 0.05, \quad \mathbb{P}(C) = 0.1, \quad \text{and} \quad \mathbb{P}(C^c) = 0.99. \]

Then

\[ \mathbb{P}(C|P) = \frac{\mathbb{P}(P|C)\mathbb{P}(C)}{\mathbb{P}(P|C)\mathbb{P}(C) + \mathbb{P}(P|C^c)\mathbb{P}(C^c)} = \frac{(0.9)(0.01)}{(0.9)(0.01) + (0.05)(0.99)}. \]