SHOW ALL WORK TO QUALIFY FOR FULL CREDIT.
Maximum possible score: 100 Points

1. **35 points** A mathematics department consists of
   - 15 full professors
   - 10 associate professors
   - 5 assistant professors.

   A committee of 6 is selected at random from the faculty of the department.
   (a) **5 points** What is the probability that the committee will consist of 1 full professor, 2 associate professors, and 3 assistant professors?
   (b) **10 points** What is the probability that the committee will consist of at least 3 full professors?
   (c) **10 points** What is the probability that the committee will consist of exactly one assistant professor and at most one full professors?
   (d) **10 points** What is the probability that the committee will consist of exactly one assistant professor given that it has at most one full professors?

2. **25 points** There are 6 balls labelled A through F in a box. We pick them out of the box one by one.
   (a) **5 points** What is the probability that the third ball is ball $B$?
   (b) **10 points** What is the probability that the third ball is ball $B$ and the fourth ball is ball $A$?
   (c) **10 points** What is the probability that the fourth ball is $A$ given that the third ball is $B$?

3. **40 points** Suppose that we select a hand of five cards out of a standard deck of 52 cards. We assume that aces are high cards.
   (a) **10 points** What is the probability that all of the cards are greater than 5?
   (b) **15 points** What is the probability that at least two of the cards are greater than 10?
   (c) **15 points** What is the probability that all of the cards are greater than 5 given that at least two of the cards are greater than 10?
1. Set $q \overset{\text{def}}{=} 1/(\binom{30}{6})$.
(a) $15\binom{10}{2}\binom{5}{3}q$.
(b) $1 - \mathbb{P}\{0,1,\text{ or } 2\text{ full professors}\} = 1 - \left\{ \binom{15}{6} + 15\binom{15}{5} + \binom{15}{2}\binom{15}{4} \right\} q$.
(c) $\left\{ \binom{5}{10} + 5 \cdot \binom{10}{4} \right\} q$.
(d) $\frac{5\binom{10}{5} + 5 \cdot 15\binom{10}{4}}{\binom{10}{6} + 15\binom{15}{5}}$.

2. (a) $\binom{5}{2}/\binom{6}{3} = \frac{1}{6}$.
(b) $\binom{1}{2}/\binom{6}{4} = \frac{1}{30}$.
(c) $\binom{1/30}{1/6} = \frac{1}{5}$.

3. Sequence: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. A set $q \overset{\text{def}}{=} 1/(\binom{52}{5})$.
(a) $\binom{36}{5}q$
(b) $1 - \mathbb{P}\{\text{no card is greater than 10}\} - \mathbb{P}\{1\text{ card is greater than 10}\}$
$= 1 - \mathbb{P}\{\text{all cards are 10 or less}\}$
$- \mathbb{P}\{1\text{ card is greater than 10, 4 cards are 10 or less}\}$
$= 1 - \binom{36}{5}q - 16\binom{36}{4}q$.
(c) $\mathbb{P}\{\text{all cards greater than 5 and at least two cards greater than 10}\}$
$= \mathbb{P}\{\text{all cards greater than 5}\}$
$- \mathbb{P}\{\text{all cards greater than 5 and no cards greater than 10}\}$
$- \mathbb{P}\{\text{all cards greater than 5, one card greater than 10, and 4 cards 10 or less}\}$
$= \mathbb{P}\{\text{all cards greater than 5}\}$
$- \mathbb{P}\{\text{all cards in 6,7,8,9,10}\}$
$- \mathbb{P}\{\text{4 cards in 6,7,8,9,10, and 1 card in J,Q,K,A}\}$
$= \binom{36}{5}q - \binom{20}{5}q - 16\binom{20}{4}q$. 

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Final answer is
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\frac{36 \choose 5 - 20 \choose 5 - 16 \cdot 20 \choose 4}{1 - 36 \choose 5 - 16 \cdot 36 \choose 4}.
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