1. **10 points** Start with a simulation of a random variable $X$ which is uniformly distributed on $(0,1)$. Make a transformation to define a second random variable $Y$ with cumulative distribution function

$$F_Y(t) \overset{\text{def}}{=} \begin{cases} 
0 & \text{if } t < 0 \\
 t & \text{if } 0 \leq t < \frac{1}{2} \\
 t + \frac{1}{4} & \text{if } \frac{1}{2} \leq t < \frac{3}{4} \\
 1 & \text{if } t \geq \frac{3}{4} 
\end{cases}$$

Your program should be able to generate a list of samples from the variable $Y$. Please also do a frequency table. Perhaps you might start by graphing $F_Y$.

2. **10 points** Simulate a Gaussian random variable with mean zero and variance 1 (note: start with uniform random variables). Using this, simulate a Gaussian random variable with mean 10 and variance 4.

3. **10 points** Simulate two independent standard Gaussian random variables $(X_1, X_2)$. Use these to then simulate a pair $(Y_1, Y_2)$ of Gaussian random variables such that

$$\mathbb{E}[Y_1] = 2 \quad \text{and} \quad \mathbb{E}[Y_2] = -5,$$

$$\mathbb{E}[(Y_1 - 2)^2] = 10, \quad \mathbb{E}[(Y_2 + 5)^2] = 4, \quad \text{and} \quad \mathbb{E}[(Y_1 - 2)(Y_2 + 5)] = 4.$$

You may want to think of $Y$ as an affine transformation of $X$; i.e.,

$$
\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = A \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + b
$$

for some $2 \times 2$ matrix $A$ and some column vector $b$. 