We will investigate the integrals

\[ \int_{x=-\infty}^{\infty} x^n e^{-x^2/2} \sqrt{2\pi} \, dx. \]  

(1)

1. **10 points** First, complete the square in the exponent to evaluate

\[ \varphi(\theta) \overset{\text{def}}{=} \int_{x=-\infty}^{\infty} e^{\theta x} x^{-x^2/2} \sqrt{2\pi} \, dx. \]  

(2)

for all \( \theta \in \mathbb{R} \). Recall that

\[ \int_{x=-\infty}^{\infty} e^{-x^2/2} \sqrt{2\pi} \, dx = 1. \]

You should get a nice expression.

2. **10 points** Recalling the Taylor series for \( e^x \), write a series expression for \( \varphi \) (with respect to the variable \( \theta \)).

3. **10 points** Recalling the Taylor series for \( e^x \), write a series expression for \( e^{\theta x} \) and substitute it in (2); you should get a series expression for \( \varphi \) which involves powers of \( \theta \) and integrals like in (1).

4. **10 points** By matching coefficients, compute all integrals of the form (1).