

Math 130, Section &E4, Fall 2001
HW3, Due December 5

We will investigate the integrals

$$\int_{x=-\infty}^{\infty} x^n \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx. \quad (1)$$

1. 10 points First, complete the square in the exponent to evaluate

$$\varphi(\theta) \stackrel{\text{def}}{=} \int_{x=-\infty}^{\infty} e^{\theta x} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx. \quad (2)$$

for all $\theta \in \mathbb{R}$. Recall that

$$\int_{x=-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx = 1.$$

You should get a nice expression.

2. 10 points Recalling the Taylor series for e^x , write a series expression for φ (with respect to the variable θ).
3. 10 points Recalling the Taylor series for e^x , write a series expression for $e^{\theta x}$ and substitute it in (2); you should get a series expression for φ which involves powers of θ and integrals like in (1).
4. 10 points By matching coefficients, compute all integrals of the form (1).