Math 130, Section &E4, Fall 2001
HW2, Due October 10

We will investigate the asymptotics of $n!$ for n large.

1. [10 points] Using the integral test, find upper and lower bounds for $\sum_{j=1}^{n} \ln j$ (note that $\sum_{j=1}^{\infty} \ln j$ diverges; use the ideas of the integral test to understand the *rate* at which it diverges).

2. [10 points] Identify
$$\lim_{n \nearrow \infty} \left\{ \frac{1}{n} \ln n! - \ln n \right\}$$

3. [10 points] On a different note, consider the integral
$$I(p) \stackrel{\text{def}}{=} \int_0^{\infty} x^p e^{-x} dx$$
for all $p > -1$.
   (a) [3 points] Verify that $I(1) = 1$
   (b) [4 points] Verify via integration by parts that $I(p) = pI(p-1)$.
   (c) [3 points] Argue that $I(n) = n!$ for all nonnegative integers n.