Math 130, Section &E4, Fall 2001
HW1, Due September 14

Suppose that a function f is given by

$$f(x) = \sum_{n=0}^{N} a_n \cos nx + \sum_{n=1}^{N} b_n \sin nx$$

for all $0 \le x \le 2\pi$.

1. $\boxed{\text{10 points}}$ Show that

$$\int_0^{2\pi} (f(x))^2\, dx = 2\pi a_0^2 + \pi \sum_{n=1}^{N} a_n^2 + \pi \sum_{n=1}^{N} b_n^2.$$

2. $\boxed{\text{10 points}}$ Show that

$$\int_0^{2\pi} \left(\frac{df}{dx}(x) \right)^2 dx = \pi \sum_{n=1}^{N} n^2 a_n^2 + \pi \sum_{n=1}^{N} n^2 b_n^2.$$

3. $\boxed{\text{10 points}}$ Suppose now that $a_0 = 0$. Define

$$F(x) \stackrel{\text{def}}{=} \int_0^x f(z) dz$$

for all $0 \le x \le 2\pi$. Show that

$$\int_0^{2\pi} (F(x))^2\, dx = \pi \sum_{n=1}^{N} \frac{a_n^2}{n^2} + \pi \sum_{n=1}^{N} \frac{b_n^2}{n^2}.$$