Math 130, Section &E4, Fall 2001
Final, December 10

SHOW ALL WORK TO QUALIFY FOR FULL CREDIT.
MAXIMUM POSSIBLE SCORE: 130 POINTS

1. [20 points] Set $a_n \stackrel{\text{def}}{=} \ln(n^2+10) - 2\ln n$ for all $n \geq 1$.
   (a) [10 points] Does $\lim_{n\nearrow\infty} a_n$ exists? If so, what is it?
   (b) [10 points] Does $\sum_{n=1}^{\infty} a_n$ exist?

2. [20 points] What is the interval of convergence of the series
$$\sum_{n=0}^{\infty} \frac{4^n(x-3)^{2n}}{\sqrt{n+3}}?$$

3. [20 points] Find the MacLaurin series (i.e., the Taylor series about $x=0$) of the functions
   (a) [10 points] $f(x) = e^x$.
   (b) [10 points] $f(x) \stackrel{\text{def}}{=} (x-4)^2$.

4. [20 points] Define the function $f(x) \stackrel{\text{def}}{=} xe^{2x}$.
   (a) [10 points] Recalling the MacLaurin series for e^x (see the previous question) give a power series (about $x=0$) for f.
   (b) [10 points] Compute $f^{(10)}(0)$.

5. [20 points] Compute the following integrals
   (a) [10 points] $\int_2^4 \frac{\ln(\ln x)}{x} dx$.
   (b) [10 points] $\int_0^1 \frac{x^2}{(4-x^2)^{3/2}} dx$.

6. [10 points] Consider the integral
$$\int_1^{\infty} \frac{e^{1/x}}{x^2} dx.$$
   Does the integral converge or diverge? If it converges, evaluate it.

7. [20 points] Consider the curve $r = 5\cos\theta$ for $0 \leq \theta \leq \pi/4$.
   (a) [10 points] Compute the area bounded by this curve and the lines $\theta = 0$ and $\theta = \pi/4$.
   (b) [10 points] Compute the arc length of this curve.