PREFACE

This is a text for a first course in real variables. The subject matter is fundamental for more advanced mathematical work, specifically in the areas of complex variables, measure theory, differential equations, functional analysis, and probability. In addition, many students of engineering, physics, and economics find that they need to know real analysis in order to cope with the professional literature in their fields. Standard mathematical writing, with its emphasis on formalism and abstraction, tends to create barriers to learning and focus on minor technical details at the expense of intuition. On the other hand, a certain amount of abstraction is unavoidable if one is to give a sound and coherent presentation. This book attempts to strike a balance that will reach the widest audience possible without sacrificing precision. (My original training was in electrical engineering, and I later became a mathematician. I seem to be able to reach students whose prior exposure to abstract reasoning is limited.)

The most important skill a student must develop in order to learn any area of mathematics is the ability to think intuitively, and a text should encourage this process–not hinder it. I find it useful to consider concrete examples that have all the features of the general case under consideration, to draw diagrams whenever appropriate, and to give geometric or physical interpretation of results. I rely especially on one of the most useful of all learning devices: the inclusion of detailed solutions to exercises. Solutions to problems are commonplace in elementary texts but quite rare (although equally valuable) at the upper division undergraduate and graduate level. This feature makes the book suitable for independent study, and further widens the audience.

I have normally covered the first seven chapters in a one-semester course. The subject matter includes metric spaces, Euclidean spaces and their basic topological properties, sequences and series of real numbers, continuous functions, differentiation, Riemann–Stieltjes integration, and uniform convergence and applications. Chapters 8 and 9 contain additional topological results, and various sections might be assigned in connection with special student projects or serve as a nice introduction to a full course in general topology.

The subject matter covered in basic courses in real variables is standard, almost canonical, but there is still room for innovation in the presentation. I have taken Cantor's Nested Set Property, rather than Cauchy completeness or least upper bound completeness, as the basic topological axiom for the real numbers. This allows a significant simplification of the proofs of the basic topological properties of R^p.