Tables

Common Density Functions and Their Properties

Type	Density	Parameters
Uniform on $[a, b]$	$\frac{1}{b-a}, \quad a \le x \le b$	a, b real, $a < b$
Normal	$\frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$	μ real, $\sigma > 0$
Gamma	$\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}, \quad x \ge 0$	$\alpha, \beta > 0$
Beta	$\frac{x^{r-1}(1-x)^{s-1}}{\beta(r, s)}, \quad 0 \le x \le 1$	$r, s > 0$
Exponential (= gamma with $\alpha = 1, \beta = 1/\lambda$)	$\lambda e^{-\lambda x}, \quad x \ge 0$	$\lambda > 0$
Chi-square (= gamma with $\alpha = n/2, \beta = 2$)	$\frac{1}{2^{n/2}\Gamma(n/2)} x^{(n/2)-1}e^{-x/2}, \quad x \ge 0$	$n = 1, 2, \ldots$
t	$\frac{\Gamma[(n+1)/2]}{\sqrt{n\pi}\,\Gamma(n/2)} \frac{1}{(1 + x^2/n)^{(n+1)/2}}$	$n = 1, 2, \ldots$
F	$\frac{(m/n)^{m/2}}{\beta(m/2, n/2)} \frac{x^{(m/2)-1}}{(1 + (m/n)x)^{(m+n)/2}}, \quad x \ge 0$	$m, n = 1, 2, \ldots$
Cauchy	$\frac{\theta}{\pi(x^2 + \theta^2)}$	$\theta > 0$

Common Density Functions (continued)

Type	Mean	Variance	Generalized Characteristic Function (If Easily Computable)
Uniform on $[a, b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{1}{b-a}\frac{e^{-sa}-e^{-sb}}{s}$, all s
Normal	μ	σ^2	$e^{-s\mu}e^{s^2\sigma^2/2}$, all s
Gamma	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1/\beta}{s+1/\beta}\right)^\alpha$, $\operatorname{Re} s > -1/\beta$
Beta	$\frac{r}{r+s}$	$\frac{rs}{(r+s)^2(r+s+1)}$	
Exponential	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{s+\lambda}$, $\operatorname{Re} s > -\lambda$
Chi-square	n	$2n$	$(2s+1)^{-n/2}$, $\operatorname{Re} s > -1/2$
t	0 if $n > 1$; does not exist if $n = 1$	$\frac{n}{n-2}$ if $n > 2$; ∞ if $n = 2$	
F	$\frac{n}{n-2}$ if $n > 2$; ∞ if $n = 1$ or 2	$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ if $n > 4$; ∞ if $n = 3$ or 4	
Cauchy	Does not exist	Does not exist	$e^{-\theta\|u\|}$, $s = iu$, u real

Common Probability Functions and Their Properties

Type	Probability $p(k)$	Parameters
Discrete uniform	$\frac{1}{N}$, $k = 1, 2, \ldots, N$	$N = 1, 2, \ldots$
Bernoulli	$p(1) = p$ $p(0) = q$	$0 \le p \le 1, q = 1 - p$
Binomial	$\binom{n}{k}p^k q^{n-k}$, $k = 0, 1, \ldots, n$	$0 \le p \le 1, q = 1 - p$, $n = 1, 2, \ldots$
Poisson	$e^{-\lambda}\lambda^k/k!$, $k = 0, 1, \ldots$	$\lambda > 0$
Geometric	$q^{k-1}p$, $k = 1, 2, \ldots$	$0 < p \le 1, q = 1 - p$
Negative binomial	$\binom{k-1}{r-1}p^r q^{k-r} = \binom{-r}{k-r}p^r(-q)^{k-r}$, $k = r, r+1, \ldots$	$0 < p \le 1, q = 1 - p$, $r = 1, 2, \ldots$

Common Probability Functions (continued)

Type	Mean	Variance	Generalized Characteristic Function
Discrete uniform	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$	$\frac{e^{-s}(1-e^{-sN})}{N(1-e^{-s})}$, all s
Bernoulli	p	$p(1-p)$	$q+pe^{-s}$, all s
Binomial	np	$np(1-p)$	$(q+pe^{-s})^n$, all s
Poisson	λ	λ	$\exp[\lambda(e^{-s}-1)]$, all s
Geometric	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^{-s}}{1-qe^{-s}}$, $\lvert qe^{-s}\rvert < 1$
Negative binomial	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pe^{-s}}{1-qe^{-s}}\right)^r$, $\lvert qe^{-s}\rvert < 1$

Selected Values of the Standard Normal Distribution Function

$F(x) = (2\pi)^{-1/2}\int_{-\infty}^{x} e^{-t^2/2}\,dt$, $\quad F(-x) = 1 - F(x)$

x	$F(x)$	x	$F(x)$	x	$F(x)$	x	$F(x)$
.0	.500	.9	.816	1.64	.950	2.33	.990
.1	.540	1.0	.841	1.7	.955	2.4	.992
.2	.579	1.1	.864	1.8	.964	2.5	.994
.3	.618	1.2	.885	1.9	.971	2.6	.995
.4	.655	1.28	.900	1.96	.975	2.7	.996
.5	.691	1.3	.903	2.0	.977	2.8	.997
.6	.726	1.4	.919	2.1	.982	2.9	.998
.7	.758	1.5	.933	2.2	.986	3.0	.999
.8	.788	1.6	.945	2.3	.989		

A Brief Bibliography

An excellent source of examples and applications of basic probability is the classic work of W. Feller, *Introduction to Probability Theory and Its Applications* (John Wiley, Vol. 1, 1950; Vol. 2, 1966). Another good source is *Modern Probability Theory* by E. Parzen (John Wiley, 1960).

Many properties of Markov chains and other stochastic processes are given in *A First Course in Stochastic Processes* by S. Karlin (Academic Press, 1966). A. Papoulis' *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill, 1967) is a treatment of stochastic processes that is directed toward engineers.

A comprehensive treatment of basic statistics is given in *Introduction to Mathematical Statistics* by R. Hogg and A. Craig (Macmillan, 1965). A more advanced work emphasizing the decision-theory point of view is *Mathematical Statistics, A Decision Theoretic Approach* by T. Ferguson (Academic Press, 1967).

The student who wishes to take more advanced work in probability will need a course in measure theory. H. Royden's *Real Analysis* (Macmillan, 1963) is a popular text for such a course. For those with a measure theory background, J. Lamperti's *Probability* (W. A. Benjamin, 1966) gives the flavor of modern probability theory in a relatively light and informal way. A more systematic account is given in *Probability* by L. Breiman (Addison-Wesley, 1968).