Index

Absolutely continuous random variable, 53
Absolutely continuous random vector, 72
Actions, set of, 242
Admissible and inadmissible tests, 247
Admissible risk points, 248
Alternative, simple and composite, 243
Average value, see Expectation

Bayes estimate, 260
with constant risk, 262
with quadratic loss function, 260
Bayes risk, 244, 260
Bayes test, 244
Bayes’ theorem, 36, 150
Bernoulli distribution, see Distribution
Bernoulli trials, 28, 38, 58, 128, 151, 175,
177, 187, 190, 195, 207, 215
generalized, 29
see also Distribution, binomial
Beta distribution, see Distribution
Beta function, 133, 261
Binomial distribution, see Distribution
Boolean algebra, 3ff
Borel-Cantelli lemma, 205
second, 209
Borel measurable function, 83
Borel sets, 47, 50
Bose-Einstein assumption, 21

Cauchy distribution, see Distribution
Central limit theorem, 169ff, 171
Characteristic function(s), 154ff

correspondence, theorem for, 156

properties of, 166ff

of a random vector, 279
Chebyshev’s inequality, 126, 127, 129, 206, 208
Coin tossing, see Bernoulli trials; Distribution,
binomial

Combinatorial problems, 15ff
fallacies in, 39ff
multiple counting in, 22
Complement of an event, 4
Conditional density, 136, 148
Conditional distribution function, 139, 140, 148

Conditional expectation, 140ff
Conditional probability, 33ff, 130ff
Conditional probability function, 98, 142
Confidence coefficient, 276
Confidence interval, 276
Confidence set, 278
Continuous random variable, 69
Convergence, almost surely (almost everywhere), 204–206, 208, 210
in distribution, 170, 171, 175, 176
in probability, 171, 175, 176, 205, 208, 210

Convex function, 262
Convexity of the risk set, 248
Convolution theorem, 164
Correlation, 119ff
Correlation coefficient, 120
Covariance, 119
Covariance function, 203
Covariance matrix, 281
Cylinder, 180
measurable, 180

Decision function, 242, 243
nonrandomized, 242
Decision scheme, 151
DeMorgan laws, 7, 9, 11
Density function(s), 53
INDEX

conditional, 136, 148
joint, 70ff, 181
marginal, 78
Difference equation, 24, 39, 182, 186, 195
characteristic equation of, 183
Discrete probability space, 15
Discrete random variables, 51, 95ff
Disjoint events, 5
Distribution, 95
Bernoulli, 256, 264, 266, 269, 272
beta, 260, 268
binomial, 29, 32, 95, 97–99, 113, 122, 141, 176, 256, 258, 260, 264, 268
Cauchy, 161, 166, 264
chi-square, 165, 275, 276, 278
F, 278
gamma, 166, 267, 268
geometric, 195, 196
hypergeometric, 33, 256
multidimensional Gaussian (joint Gaussian), 279ff
negative binomial, 196, 264, 268, 272
Poisson, 96–99, 114, 152, 163, 169, 197, 198, 200, 202, 256, 264, 266, 268, 270, 272
r, 277, 278
uniform, 54, 73, 76, 84, 92, 93, 113, 118, 141, 149, 150–152, 165, 208, 257, 263, 264, 267, 271, 272
Distribution function(s), 52
conditional, 139, 140, 148
joint, 72
properties of, 66ff
Dominated convergence theorem, 231
Essentially constant random variable, 85, 115
Estimate, 258
Bayes, 260
with constant risk, 262
inadmissible, 272
maximum likelihood, 258
minimax, 262
randomized, 258, 263
risk function of, 261
unbiased, 268
uniformly minimum variance unbiased (UMVUE), 269
Estimation, 152, 242, 243, 258ff
Event(s), 2, 11
algebra of, 3ff
complement of, 4
contracting sequence of, 67
exhaustive, 35
expanding sequence of, 66
impossible, 3, 55
independent, 26, 27
intersection of, 4
mutually exclusive (disjoint), 5
union of, 4
upper and lower limits of sequence of, 204, 209
sure (certain), 3
Expectation, 100ff
conditional, 140ff
general definition of, 103
properties of, 114ff
Exponential distribution, see Distribution
F distribution, see Distribution
Factorization theorem, 266
Fatou’s lemma, 230
Fermi-Dirac assumption, 20
Fourier series, 167
Fourier transform, 155
Gambler’s ruin problem, 182ff, 235
Gamma distribution, see Distribution
Gamma function, 109, 133
Gaussian distribution, see Distribution, normal
Generating function, 169, 191ff
moments obtained from, 192
Geometric distribution, see Distribution
Hypergeometric distribution, see Distribution
Hypothesis, 243ff
a priori probability of, 244
composite, 243
compound, 243
simple, 243
Hypothesis testing, 151, 242, 243ff
fundamental theorem of, 246
see also Test
Independence, 25ff
INDEX

Independence of sample mean and variance in normal sampling, 274
Independent events, 26, 27
Independent random variables, 80
Indicators, 122ff
Intersection of events, 4

Jensen’s inequality, 262
Joint characteristic function, 279
Joint density function, 70ff, 181
Joint distribution function, 72
Joint probability function, 76, 96, 180, 181

Kolmogorov extension theorem, 180

Laplace transform, 155
properties of, 156, 157
Lattice distribution, 169
Law of large numbers, strong, 129, 203, 206, 207
weak, 128, 169, 171, 207
Lebesgue integral, 114
Level of a test, 246
Liapounov condition, 175
Likelihood ratio, 245
test (LRT), 245
Limit inferior (lower limit), 204, 209
Limit superior (upper limit), 204, 209
Linearly dependent random variables, 121, 281
Loss function (cost function), 242
quadartic, 260

Marginal densities, 78
Markov chain(s), 211ff
closed sets of, 224
cyclically moving subclasses of, 227
definition of, 214
equivalence classes of states of, 223
first entrance theorem for, 220
initial distribution of, 213
limiting probabilities of, 230ff
state distribution of, 214
state space of, 213
states of, 220ff
aperiodic, periodic, 229
essential, 229
mean recurrence time of, 230
period of, 226–229
recurrent (persistent), 221ff
recurrent null, 233
recurrent positive, 233
transient, 221ff
stationary distribution for, 236
steady state distribution for, 215, 237
stopping time for, 217
strong Markov property of, 219
transition matrix of, 214
n-step, 214
transition probabilities of, 214
Maximum likelihood estimate, 258
Maxwell-Boltzmann assumption, 20
Median, 112
Mean, see Expectation
Minimax estimate, 262
Minimax test, 250
Moment-generating property of characteristic functions, 167, 168
Moments, 107
central, 108
joint, 119
obtained from generating functions, 192
Multinomial probability function, 30, 98
Mutually exclusive events, 5
Negative binomial distribution, see Distribution
Negative part of a random variable, 104
Neyman-Pearson lemma, 246
Normal distribution, see Distribution

Observable, 242
Order statistics, 91

Partial fraction expansion, 159
Poisson distribution, see Distribution
Poisson random process, 196ff
Poker, 19, 23, 40
Positive part of a random variable, 104
Power function of a test, 253
Power of a test, 246
Probability, 10ff
a posteriori, 36
classical definition of, 1, 13, 16
conditional, 33ff
frequency definition of, 2, 13
Probability function, 51
conditional, 98, 142
joint, 76, 96, 180, 181
Probability measure(s), 12
consistent, 180
Probability space, 12
discrete, 15

Queueing, 216

Random process, 196
Random telegraph signal, 203
Random variable(s), 46ff
  absolutely continuous, 53
  central moments of, 108
  characteristic function of, 154ff
  classification of, 51ff
  continuous, 69
  definition of, 48, 50
  degenerate (essentially constant), 85, 115
  density function of, 53
  discrete, 51, 95ff
  functions of, 58ff, 84, 85ff, 94
  generating function of, 192ff
  independent, 80
  infinite sequences of, 178ff
  linearly dependent, 121, 281
  moments of, 107
  positive and negative parts of, 104
  probability function of, 51
  simple, 101
Random vector, 72
  absolutely continuous, 72
Random walk, 184ff
  combinatorial approach to, 186ff
  simple, 184
  with absorbing barriers, 184, 185, 215, 228, 240
  with no barriers, 185, 186–191, 193–195, 215, 228, 240
  average length of time required to return to 0 in, 186, 191, 195
  distribution of first return to 0 in, 189
  first passage times in, 190
  probability of eventual return to 0 in, 185
  with reflecting barriers, 229, 240
Rao-Blackwell theorem, 263
Recurrent (persistent) states of a Markov chain, 221
Reflection principle, 188
Renewal theorem, 235
Risk function, 261
Risk set, 248

Sample mean, 259, 274
Sample space, 2

Sample variance, 259, 274
Samples, 16ff
  ordered, with replacement, 16
  without replacement, 16
  unordered, with replacement, 18
  without replacement, 17
Sampling from a normal population, 274
Schwarz inequality, 119, 121, 207
Sigma field, 11
Simple random variable, 101
Size of a test, 246
Standard deviation, 108
States of nature, 241
Statistic, for a random variable, 265
  complete, 269
  sufficient, 265
Statistical decision model, 241
Statistics, 241ff
Stirling's formula, 43, 191
Stochastic matrix, 212
Stochastic process, 196
Stopping times, 217ff
Strong law of large numbers, 129, 203, 206, 207
Strong Markov property, 219

t Distribution, see Distribution
Test, 243
  acceptance region of, 278
  admissible and inadmissible, 247
  Bayes, 244
  level of, 246
  likelihood ratio (LRT), 245
  minimax, 250
  power of, 246
  power function of, 253
  rejection region (critical region) for, 243
  risk set of, 248
  size of, 246
  type 1 and type 2 errors of, 243
  uniformly most powerful (UMP), 253
Total expectation, theorem of, 144, 149, 152, 153
Total probability, theorem of, 35, 90, 130, 132, 134, 150, 182, 214
Transient states of a Markov chain, 221
Uniform distribution, see Distribution
Uniformly most powerful (UMP) test, 253
Union of events, 4
Unit step function, 157
INDEX

Variance, 108, 155–118
Venn diagrams, 4

Weak law of large numbers, 128, 169, 171, 207