

BASIC PROBABILITY THEORY

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Preface

This book has been written for a first course in probability and was developed from lectures given at the University of Illinois during the last five years. Most of the students have been juniors, seniors, and beginning graduates, from the fields of mathematics, engineering and physics. The only formal prerequisite is calculus, but an additional degree of mathematical maturity may be helpful.

In talking about nondiscrete probability spaces, it is difficult to avoid measure-theoretic concepts. However, to develop extensive formal machinery from measure theory before going into probability (as is done in most graduate programs in mathematics) would be inappropriate for the particular audience to whom the book is addressed. Thus I have tried to suggest, when possible, the underlying measure-theoretic ideas, while emphasizing the probabilistic way of thinking, which is likely to be quite novel to anyone studying this subject for the first time.

The major field of application considered in the book is statistics (Chapter 8). In addition, some of the problems suggest connections with the physical sciences. Chapters 1 to 5, and Chapter 8 will serve as the basis for a one-semester or a two-quarter course covering both probability and statistics. If probability alone is to be considered, Chapter 8 may be replaced by Chapter 6 and Chapter 7, as time permits. An asterisk before a section or a problem indicates material that I have normally omitted (without loss of continuity), either because it involves subject matter that many of the students have not been exposed to (for example, complex variables) or because it represents too concentrated a dosage of abstraction.

A word to the instructor about notation. In the most popular terminology, $P\{X \leq x\}$ is written for the probability that the random variable X assumes a value less than or equal to the number x . I tried this once in my class, and I found that as the semester progressed, the capital X tended to become smaller in the students' written work, and the small x larger. The following semester, I switched to the letter R for random variable, and this notation is used throughout the book.

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Fairly detailed solutions to some of the problems (and numerical answers to others) are given at the end of the book.

I hope that the book will provide an introduction to more advanced courses in probability and real analysis and that it makes the abstract ideas to be encountered later more meaningful. I also hope that nonmathematics majors who come in contact with probability theory in their own areas find the book useful. A brief list of references, suitable for future study, is given at the end of the book.

I am grateful to the many students and colleagues who have influenced my own understanding of probability theory and thus contributed to this book.

I also thank Mrs. Dee Keel for her superb typing, and the staff of Wiley for its continuing interest and assistance.

Urbana, Illinois, 1969

Robert B. Ash

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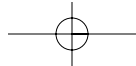
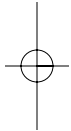
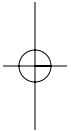
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