1. Let \( f(x) = \begin{cases} x^2 \sin \left( \frac{1}{x} \right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \).

Show that \( f(x) \) is differentiable at every real number and compute its derivative. (Hint: First compute the derivative when \( x \neq 0 \). Then consider \( x = 0 \) and use the definition.)

2. Find the derivatives of the following functions. Use prime notation when a variable is indicated on the left-hand side; use Leibniz notation where no variable is indicated on the left-hand side.
   (a) \( y = \frac{\sin t}{1 + \tan t} \)
   (b) \( g(v) = \frac{e^v}{\sqrt{v}} \)
   (c) \( u = \frac{x}{x^2 - 1} \)
   (d) \( g(t) = t \sin \pi t \)
   (e) \( h(u) = u e^u \cot u \)
   (f) \( p(\theta) = \frac{\tan \theta}{\csc \theta + \sec \theta} \)

3. A function is called odd if \( f(-x) = -f(x) \) for all \( x \) and even if \( f(-x) = f(x) \) for all \( x \).
   (a) Sketch the graph of an odd function; sketch the graph of an even function.
   (b) Show that \( x^2 \) is even. Is \( x^{2n} \) even for all \( n \), odd for all \( n \), or does it depend on \( n \)? Justify your answer. What about \( x^{2n+1} \)?
   (c) Show that the derivative of an arbitrary odd function is even and that the derivative of an arbitrary even function is odd.
   (d) Show that if \( f(x) \) is odd, then \( f(0) = 0 \). Is the same true for even functions?
   (e) Give an example of a function that is both odd and even. Prove that your example is the only function that is both odd and even.