Math 221 AD2: Worksheet 14

October 31, 2018

1. Use Newton’s method to approximate the positive root of $3 \sin x = x$ correct to two decimal places.

2. What is an antiderivative of $\cos x$? Can you guess an antiderivative of $\cos(2x)$? What about $\cos(x/2)$? Generalize this to find an antiderivative of $\cos(ax)$ for any real number $a$. Find an antiderivative for $\sin(ax)$ for any real number $a$.

3. (a) Write $1 + 2 + 3 + 4 + \cdots + 100$ in $\Sigma$ notation. Do the same for $1 + 2 + 3 + \cdots + n$. What is this equal to (as a function of $n$)?

(b) Write $2 + 4 + 6 + 8 + \cdots + 200$ in $\Sigma$ notation. Do the same for $2 + 4 + 6 + 8 + \cdots + 2n$. What is this equal to (as a function of $n$)?

(c) Write $1 + 4 + 9 + 16 + \cdots + 100$ in $\Sigma$ notation. Do the same for $1 + 4 + 9 + \cdots + n^2$. What is this equal to (as a function of $n$)?

(d) Write $1 + 2 + 4 + 8 + 16 + \cdots + 1024$ in $\Sigma$ notation.

4. Given the function $f(x) = x^2 - x$, we want to estimate, and then actually compute, the area underneath $f$ between $x = 1$ and $x = 5$. We will use a Riemann sum with right endpoints.

(a) First, divide $[1,5]$ into two subintervals of equal length, $[1,3]$ and $[3,5]$, and approximate the area underneath $f$ by $2f(3) + 2f(5)$. Sketch a picture illustrating this approximation. Is your approximation an overestimate or an underestimate of the actual area?

(b) Now, divide $[1,5]$ into four subintervals of equal length, $[1,2]$, $[2,3]$, $[3,4]$, and $[4,5]$, and approximate the area underneath $f$ by $f(2) + f(3) + f(4) + f(5)$. Sketch a picture illustrating this approximation. Is your approximation an overestimate or an underestimate of the actual area?

(c) Now, divide $[1,5]$ into $n$ subintervals and write a Riemann sum using right endpoints. This is a generalization of parts (a) and (b). Since $f(x) = x^2 - x$ is integrable on $[1,5]$ (this follows from the fact that it is continuous on $[1,5]$), the area underneath it is actually equal to the limit of the sum you just wrote as $n$ tends to infinity. Compute this limit.

(d) Redraw your pictures from parts (a) and (b) using left endpoints instead of right endpoints. Would the sum have been an overestimate or an underestimate?

5. Which of the following is true? Justify your answer.

A. $\int_{0}^{1} \sqrt{1 + x^2} \, dx \geq \int_{0}^{1} \sqrt{1 + x} \, dx$

B. $\int_{0}^{1} \sqrt{1 + x^2} \, dx \leq \int_{0}^{1} \sqrt{1 + x} \, dx$