MATH 221 :: Calculus I

Mock Exam III

Fall 2018

Name: ____________________________________________

• Show ALL your work.
• There are NO calculators allowed on this exam.
• This exam has 13 questions.
1. State the Fundamental Theorem of Calculus, parts I and II.

2. True or False:

\[ \int_{-1}^{1} \frac{1}{x^2} \, dx = -\frac{1}{x} \bigg|_{-1}^{1} = -1 - 1 = -2. \]

3. Starting with the guess of \( x_1 = 1 \), use Newton’s Method to find the following two approximations \( x_2 \) and \( x_3 \) of the root of the function \( f(x) = x^3 - 7 \).
4. Use the limit definition of the integral to evaluate the following integral.

\[
\int_{-2}^{0} x^2 + x \, dx
\]
5. Let \( g(x) = \int_{(2x+1)^3}^{6} \sin \left( t^4 + 8 \right) \, dt \). Determine \( g'(x) \).

6. A function \( f(x) \) has derivative \( f'(x) = 20x^9 + 8e^x + \frac{10}{\sqrt{1-x^2}} + 12 \sec(x) \tan(x) \). Find a formula for \( f(x) \) given that its graph goes through the point \((0, 50)\).

7. At time \( t \) days, the valuation of a startup is growing at a rate of \( 9t^2 + 4t + 10 \) dollars per day. If the startup's valuation at day \( t = 1 \) is $1000, what is the startup's valuation at day \( t = 3 \)?
8. Evaluate the definite integral. Simplify your answer.

\[ \int_{-1}^{1} 20x^5 + \frac{4x^3 + 3x^2 + 4x + 5}{x^2 + 1} \, dx \]

9. Find the indefinite integral.

\[ \int \left( \frac{x + 1}{x} \right) \sin (\ln x + x) \, dx \]
10. Find the indefinite integral.

\[ \int \frac{3e^x + 2e^{2x}}{\sqrt{1 - e^{2x}}} \, dx \]

11. If \( f(x) = \int_0^x (1 - t^2)e^{t^2} \, dt \), on what interval is \( f \) increasing?
12. Evaluate the definite integrals.

(a) \( \int_1^2 (2x + 1)^2 \, dx \)

(b) \( \int_0^{\pi/4} \tan^2 x \, dx \)

13. Find the indefinite integral.

\( \int \sqrt{\sin x} \cos^3 x \, dx \)