MATH 221 :: Calculus I

Mock Exam I

Fall 2018

Name: 

- Show **ALL** your work.
- There are **NO** calculators allowed on this exam.
- This exam has 10 questions.
1. Let \( f(x) = \frac{x - 2}{x^2 - 8x + 12} \). Circle the correct statement from the following.

   A. \( f(x) \) is differentiable at \( x = 2 \) because it is continuous at \( x = 2 \).

   B. \( f(x) \) is differentiable at \( x = 2 \) because \( f'(x) = \frac{-x^2 + 4x - 4}{(x^2 - 8x + 12)^2} \) for all \( x \).

   C. \( f(x) \) is not differentiable at \( x = 2 \) because \( \lim_{x \to 2^-} \frac{-x^2 + 4x - 4}{(x^2 - 8x + 12)^2} \neq \lim_{x \to 2^+} \frac{-x^2 + 4x - 4}{(x^2 - 8x + 12)^2} \).

   D. \( f(x) \) is not differentiable at \( x = 2 \) because it is not continuous at \( x = 2 \).

2. Let \( f(x) = \frac{x^3 + 2x^2 - 7x + 3}{x^2 - 9} \). Circle the correct statement from the following.

   A. \( f(x) \) has a zero in \([0, 2]\) because \( f(0) < 0 \) and \( f(2) > 0 \).

   B. \( f(x) \) has no zeroes in \([0, 2]\) because \( f(0) < 0 \) and \( f(2) < 0 \).

   C. \( f(x) \) has two zeroes in \([0, 2]\).

   D. The Intermediate Value Theorem is inconclusive because \( f(x) \) is not continuous.
3. Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of “does not exist” is not sufficient. For infinite limits you must state if it is $\infty$ or $-\infty$.

(a) \[ \lim_{x \to \infty} \frac{42 - 15e^{2x}}{6e^x + 100} \]

(b) \[ \lim_{x \to \infty} \frac{\arctan \left( \sin \left( x^4 e^x \right) \right)}{x^4} \]
(c) \[ \lim_{x \to \infty} \frac{\ln \left(2 + \frac{1}{x^2}\right)}{\ln \left(1 - e^{-x}\right)} \]

(d) \[ \lim_{x \to \ln 7} \frac{e^x - 7}{e^{2x} - 49} \]
4. Consider the function \( f(x) = -90 + 30e^{\frac{x}{3}} \). Find the equation of the tangent line at the \( x \)-intercept of \( f \).

5. Determine an equation for each horizontal asymptote on the graph of the following function. Your answer must be justified using limits.

\[
f(x) = \frac{2e^x}{e^x - 5}
\]
6. Find \( \frac{dy}{dx} \) for each of the following equations. Show all your work.

(a) \( y = \ln (x^2 2^x + 1) \)

(b) \( y = e^{\cos^3(x) + \arctan(\sqrt{x})} \)

(c) \( y = \frac{\sqrt{x^2 + 1}}{7x^2 + 3x^6} \)
7. The point $(-1, 0)$ is on the curve defined by the equation:

$$x^2 y^3 - e^{y^2} = \sqrt[3]{x} + xy$$

Find the value of $\frac{dy}{dx}$ when $x = -1$.

8. Find $\frac{dy}{dx}$, where $y = \frac{(\ln x)^x}{2^{3x+1}}$. Your final answer must be entirely in terms of $x$. 
9. Let \( f(x) = \frac{1}{\sqrt{x} + 2} \). Use the definition of a derivative as a limit to find the derivative of \( f(x) \). Show each step in your calculation and be sure to use proper terminology in each step of your proof.
10. Far off into the future on a mysterious exoplanet, an explorer tosses a marble straight into the air to test the gravity. Her heads-up-display gives her the marble’s height in meters as a function of seconds since it was thrown:

\[ h(t) = -3t^2 + 36t + 2 \]

As the mission’s intern, you must answer the following questions:

(a) What is the marble’s initial velocity?

(b) What is the maximum height the marble reaches?

(c) What is the acceleration due to gravity on that planet?