Math 221 CD1: Test 3 Review—Net Change Theorem and Average Values

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**Theorem 1.1** (Net Change Theorem).

**Definition 1.2.** The *average value* of a function $f$ on $[a, b]$ is given by

$$f_{av} =$$

**Theorem 1.3** (Mean Value Theorem for Integrals).

**Example 1.4.** At time $t$ hours, a population of bacteria is growing at a rate of $3t^2 + 2t + 50$ bacteria per hour. If the population 2000 is at time $t = 2$ hours, then what is the population at time $t = 4$ hours?

**Example 1.5.** Find the average value of $f(x) = x^2 - 2x + 3$ on $[-1, 1]$. 
Theorem 2.1 (The Substitution Rule). If \( u = g(x) \) is a differentiable function whose range is an interval \( I \) and \( f \) is continuous on \( I \), then

\[
\int f(g(x)) \, g'(x) \, dx =
\]

Example 2.2. Find the indefinite integral.

\[
\int \frac{\ln \sqrt{x}}{x} \, dx
\]

Example 2.3. Find the indefinite integral.

\[
\int \frac{3x + 2x^3}{x^3 + 16} \, dx
\]
Example 3.1. Find the indefinite integral.
\[
\int \sin^2 x \cos^2 x \, dx
\]

Example 3.2. Find the indefinite integral.
\[
\int (\cos(2x) \sec x + \sin^2 x \sec x) \, dx
\]
Example 4.1. Consider the curves $y = x - 1$ and $x = y^2 - 4y + 5$.

(a) Sketch a careful picture of these curves and their intersection points.

(b) Set up an integral(s) in terms of $y$ to compute the area between the curves.

(c) Set up an integral(s) in terms of $x$ to compute the area between the curves.

(d) Compute the area between the curves.
Example 5.1. Let $R$ be the region enclosed by the curves $y = \ln x$, $y = 0$, and $x = 3$.

(a) Sketch a careful picture of $R$.

(b) Set up but do not evaluate an integral in terms of $y$ to find the volume of the solid obtained by rotating $R$ around the line $x = 1$.

(c) Set up but do not evaluate an integral in terms of $x$ to find the volume of the solid obtained by rotating $R$ around the line $x = 1$.

Example 5.2. Set up an integral to find the volume of the solid whose base is the region bounded by the curve $y = x^2 - 4x + 5$ and the line $y = 2$, and whose cross-sections perpendicular to the $x$-axis are equilateral triangles.