1. (3 points) Evaluate the following limit.

\[
\lim_{x \to 2} \frac{e^{x-2} - x^2 + 3x - 3}{(x-2)^2}
\]

\[
= \lim_{x \to 2} \frac{e^{x-2} - x^2 + 3 \to 0}{(x-2)^2} \to 0, \text{ so L'Hôpital's Rule applies}
\]

\[
= \lim_{x \to 2} \frac{e^{x-2} - 2x + 3}{2(x-2)} \to 0, \text{ so L'Hôpital's Rule applies}
\]

\[
= \lim_{x \to 2} \frac{e^{x-2} - 2}{2}
\]

\[
= -\frac{1}{2}
\]
2. (3 points) Evaluate the following limit.

\[
\lim_{x \to \infty} \left( 7 + 3e^{2x} \right)^{3/x}
\]

Since \((7 + 3e^{2x})^{3/x} = e^{\ln \left( (7 + 3e^{2x})^{3/x} \right)}\), if \(\lim_{x \to \infty} \ln \left( (7 + 3e^{2x})^{3/x} \right)\) exists, then

\[
\lim_{x \to \infty} \left( 7 + 3e^{2x} \right)^{3/x} = e^{\lim_{x \to \infty} \ln \left( (7 + 3e^{2x})^{3/x} \right)}
\]

because \(e^x\) is continuous at all real numbers.

Then

\[
\lim_{x \to \infty} \ln \left( (7 + 3e^{2x})^{3/x} \right) = \lim_{x \to \infty} \frac{3 \ln (7 + 3e^{2x})}{x} \to \infty, \text{ so L'Hôpital's Rule applies}
\]

\[
= \lim_{x \to \infty} \frac{3}{7 + 3e^{2x}} \cdot \frac{6e^{2x}}{x} \to \infty
\]

\[
= \lim_{x \to \infty} \frac{18e^{2x}}{7 + 3e^{2x}}
\]

\[
= \lim_{x \to \infty} \frac{18}{7e^{2x} + 3} \text{ (alternatively, you could have used L'Hôpital's Rule again)}
\]

\[
= \frac{18}{3} = 6.
\]

Hence

\[
\lim_{x \to \infty} \left( 7 + 3e^{2x} \right)^{3/x} = e^6.
\]
3. (4 points) For each \( x > 0 \), a triangle is formed with vertices \((0, 0)\), \((x, 0)\) and \((x, f(x))\) where \( f(x) \) is the function given below. What is the value of \( x \) which results in the triangle of largest area?

\[
f(x) = \frac{10}{x^2 + 3x + 9}
\]

For \( x > 0 \), the area formed by the triangle with vertices \((0, 0)\), \((x, 0)\) and \((x, f(x))\) is given by

\[
A(x) = \frac{xf(x)}{2} = \frac{5x}{x^2 + 3x + 9}.
\]

This is the function we want to maximize. Then

\[
A'(x) = \frac{5(x^2 + 3x + 9) - 5x(2x + 3)}{(x^2 + 3x + 9)^2} = \frac{45 - 5x^2}{(x^2 + 3x + 9)^2}.
\]

Note that \( x^2 + 3x + 9 \geq 9 > 0 \) for \( x > 0 \), so \( A'(x) \) is never undefined. Since \( A'(x) = 0 \) if and only if \( 45 - 5x^2 = 0 \), we obtain two critical numbers \( x = 3 \) and \( x = -3 \). But \( x = -3 \) is not in the domain of \( A \), so \( x = 3 \) is the only critical number.

It remains to check that \( x = 3 \) maximizes the area. Since \( A' \) is positive on \((0, 3)\) and negative on \((3, \infty)\), \( A \) must have a global maximum at \( x = 3 \).