Math 221 (section CD1) Quiz 4 Fall 2017

Solutions

- You have 20 minutes  
- No calculators  
- Show sufficient work

1. (4 points) A cone-shaped coffee filter of radius 5 cm and height 10 cm contains water, which drips through a hole at the bottom at a constant rate of 1.5 cm³ per second. How quickly is the height of the water in the filter decreasing when its height is 6 cm?

Let \( r, h, \) and \( V \) denote the radius, height, and volume of the cone formed by the water in the filter, respectively. Note that all three quantities are changing with respect to time. By similar triangles (draw yourself a picture), we see that \( r = \frac{h}{2} \), so \( V = \pi r^2 \frac{h}{3} = \pi \frac{h^3}{12} \).

Differentiating, we then have \( \frac{dV}{dt} = \frac{\pi h^2}{4} \cdot \frac{dh}{dt} \). We are given \( \frac{dV}{dt} = -1.5 = -\frac{3}{2} \), so when \( h = 6 \),

\[
\frac{dh}{dt} = -\frac{3 \cdot 4}{2\pi(6)^2} = -\frac{1}{6\pi}.
\]

Hence the height of the water in the filter is decreasing at a rate of \( 1/6\pi \) cm/s when its height is 6 cm.
2. (3 points) Suppose that $g$ and $g'$ are differentiable everywhere and satisfy the following conditions.

- $g(6) = 10$
- $g'(6) = 5$
- $g''(6) = 3$

Use a linear approximation to estimate the following quantities. Simplify and write your answers in decimal form.

(a) $g(5.7)$

$$g(5.7) \approx g(6) + g'(6)(5.7 - 6) = 10 + 5(-0.3) = 8.5$$

(b) $g'(5.7)$

$$g'(5.7) \approx g'(6) + g''(6)(5.7 - 6) = 5 + 3(-0.3) = 4.1$$

3. (3 points) Use a linear approximation to obtain a good estimate for $(1006)^{2/3}$. Simplify and write your answer in decimal form.

Let $f(x) = x^{2/3}$ and $a = 1000$. Then $f'(x) = \frac{2}{3}x^{-1/3}$, so $f(1000) = 100$ and $f'(1000) = 1/15$. Thus

$$(1006)^{2/3} = f(1006) \approx 100 + \frac{1}{15}(1006 - 1000) = 100 + \frac{6}{15} = 100.4.$$