Solutions

• 20 minutes  
• No calculators  
• Show sufficient work  

• You may use derivative short-cut rules for problems 1abc and 2, but not for problem 3.

1. (2 points each) Using Leibniz notation (i.e., \( \frac{dy}{dx} \), \( \frac{dP}{dt} \), etc.), find derivatives for each of the following functions.

(a) \( F = \left( \sqrt[4]{\frac{4t^3}{\sqrt{t}}} \right)^4 + 13 \ln(17 + \cos(9/2)) \) (simplify your answer)

\[
F = \left( \sqrt[4]{\frac{4t^3}{\sqrt{t}}} \right)^4 + 13 \ln(17 + \cos(9/2)) = \left( 4t^{5/2} \right)^2 + 13 \ln(17 + \cos(9/2)) \\
= 16t^5 + 13 \ln(17 + \cos(9/2))
\]

Thus \( \frac{dF}{dt} = 80t^4 \).

(b) \( g = 7e^r r^3 \)

\[
\frac{dg}{dr} = 7e^r r^3 + 7e^r \cdot 3r^2 \\
= 7e^r r^3 + 21e^r r^2 \\
= 7e^r (r^3 + 3r^2)
\]

(c) \( u = \frac{8 \tan \theta}{\theta^2 + 3} \)

\[
\frac{du}{d\theta} = \frac{8(\sec^2 \theta)(\theta^2 + 3) - 2\theta \cdot 8 \tan \theta}{(\theta^2 + 3)^2} \\
= \frac{8(\sec^2 \theta)(\theta^2 + 3) - 16\theta \tan \theta}{(\theta^2 + 3)^2}
\]
2. (2 points) Find the $x$-value for each point on the graph of $h(x) = 2x^3 + 5x + 37$ where the line tangent to the curve is perpendicular to the line $x + 29y = 58$.

The slope of the tangent line to the graph of $h(x)$ is given by $h'(x) = 6x^2 + 5$. The line $x + 29y = 58$ has slope $-1/29$, so a perpendicular line must have slope $29$.

If $h'(x) = 6x^2 + 5 = 29$, then $x^2 = 4$, so $x = \pm 2$.

3. (2 points) Let $f(x) = 3x^2 - 4x$.

Use the definition of a derivative as a limit to prove that $f'(x) = 6x - 4$.

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
= \lim_{h \to 0} \frac{3(x + h)^2 - 4(x + h) - 3x^2 + 4x}{h}
= \lim_{h \to 0} \frac{3x^2 + 6xh + h^2 - 4x - 4h - 3x^2 + 4x}{h}
= \lim_{h \to 0} \frac{6xh + h^2 - 4h}{h}
= \lim_{h \to 0} (6x + h - 4)
= 6x - 4
\]