(circle your TA discussion section)

- CD1, WF 9:00-10:50, Nigel Pynn-Coates
- CD2, WF 1:00-2:50, Chris Linden
- CD3, WF 9:00-10:50, Stefan Klajbor-Goderich
- CDA, WF 8:00-8:50, Hsin-Po Wang
- CDB, WF 9:00-9:50, Hsin-Po Wang
- CDC, WF 10:00-10:50, Albert Tamazyan
- CDD, WF 11:00-11:50, Dara Zirlin
- CDE, WF 12:00-12:50, Dara Zirlin
- CDF, WF 1:00-1:50, Albert Tamazyan
- CDH, WF 3:00-3:50, Xiaolong ‘Hans’ Han
- CDI, WF 8:00-8:50, Haojian Li
- CDJ, WF 9:00-9:50, Jianting ‘Jesse’ Huang
- CDK, WF 10:00-10:50, Haojian Li
- CDL, WF 11:00-11:50, Xiaolong ‘Hans’ Han
- CDM, WF 1:00-1:50, Dana Neidinger
- CDN, WF 2:00-2:50, Ningchuan Zhang
- CDO, WF 8:00-8:50, Lan Wang
- CDP, WF 9:00-9:50, Lan Wang
- CDQ, WF 10:00-10:50, Xinghua Gao
- CDR, WF 11:00-11:50, Xinghua Gao
- CDS, WF 12:00-12:50, Jianting ‘Jesse’ Huang
- CDT, WF 1:00-1:50, Ningchuan Zhang
- CDU, WF 2:00-2:50, Dana Neidinger

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 221 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- The quiz is due at the beginning of your official discussion period on Friday (Nov 17).
- Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until the quizzes have been collected for all MATH 221 discussion sections.
1. (2 points) Evaluate $\int \frac{x^5 + x^2}{4x^6 + 1} \, dx$

\[ \int \frac{x^5}{4x^6 + 1} \, dx + \int \frac{x^2}{4x^6 + 1} \, dx \]

\[ = \frac{1}{24} \ln(4x^6 + 1) + \frac{1}{6} \arctan(2x^3) + C \]

2. (2 points) Evaluate $\int e^{2x} \sqrt{e^{2x} + 1} \, dx$

\[ u = e^{2x} + 1 \]
\[ du = 2e^{2x} \, dx \]
\[ \frac{1}{2} du = e^{2x} \, dx \]

Note: $u = e^{2x} + 1 \Rightarrow e^{2x} = u - 1$

\[ = \int \frac{(u-1)^2 \sqrt{u}}{u^{\frac{1}{2}}} \, du \]
\[ = \frac{1}{2} \int (u^\frac{3}{2} - 2u^\frac{1}{2} + 1) u^{\frac{1}{2}} \, du \]
\[ = \frac{1}{5} \left( \frac{2}{3} u^{\frac{7}{2}} - \frac{4}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} \right) + C \]
\[ = \frac{1}{7} u^{\frac{7}{2}} - \frac{2}{5} u^{\frac{5}{2}} + \frac{1}{3} u^{\frac{3}{2}} + C \]

\[ = \frac{1}{7} (e^{2x} + 1)^{\frac{7}{2}} - \frac{2}{5} (e^{2x} + 1)^{\frac{5}{2}} + \frac{1}{3} (e^{2x} + 1)^{\frac{3}{2}} + C \]
3. (2 points) Let \( f(x) = h - 4hx^2/b^2 \) where \( h \) and \( b \) represent arbitrary positive real numbers. Sketch the graph of \( f(x) \) being sure to label the \( x \)-intercepts and \( y \)-intercept. Determine the area of the finite region bounded by the graph of \( f(x) \) and the \( x \)-axis. Simplify your answer.

\[
\begin{align*}
&\text{y-intercept} \quad \text{set } x = 0 \\
&y = h - \frac{4hb(x)^2}{b^2} = h
\\
&\text{x-intercepts} \quad \text{set } y = 0 \\
&0 = h - \frac{4hbx^2}{b^2} \Rightarrow x = \pm b/2
\end{align*}
\]

\[
\text{area} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \left( h - \frac{4hrx^2}{b^2} \right) dx
\]

\[
= 2 \int_{0}^{\frac{b}{2}} \left( h - \frac{4hx^2}{b^2} \right) dx
= 2 \left[ hx - \frac{4hx^3}{3b^2} \right]_{0}^{\frac{b}{2}}
= 2 \left[ h \left( \frac{b}{2} \right) - \frac{4hb^3}{3b^2} \right] - \left[ 0 - 0 \right]
= 2 \left[ \frac{hb}{2} - \frac{4hb^3}{24b^2} \right]
= 2 \left[ \frac{hb}{2} - \frac{hb^3}{12b^2} \right]
\]

\[
\text{area} = \frac{2}{3}bh
\]

Thus, the area of a parabolic arch is \( \frac{2}{3} \text{(base)} \times \text{(height)} \).
4. (2 points) Determine the area of the finite region bounded by the graphs of \( x = 3y^2 + 18y \) and \( x = 6y \). Simplify your answer.

\[ x = 3y^2 + 18y = 3y(y + 6) \] is the parabola shown. Set \( 3y^2 + 18y = 6y \) for intersection points:

\[
\begin{align*}
3y^2 + 12y &= 0 \\
3y(y + 4) &= 0 \\
y &= 0 \Rightarrow x = 0; 0 = 0 \\
-4 \Rightarrow x = 6(-4) = -24
\end{align*}
\]

\[
\text{Area} = \int_{y_{\min}}^{y_{\max}} (x_{\text{right}} - x_{\text{left}}) \, dy
\]

\[
= \int_{-4}^{0} (6y - (3y^2 + 18)) \, dy
\]

\[
= \int_{-4}^{0} (-12y - 3y^2) \, dy
\]

\[
= (-6y^2 - y^3) \bigg|_{-4}^{0}
\]

\[
= (0 - 0) - (-6(-4)^2 - (-4)^3)
\]

\[
= 32
\]

Note: We did not integrate with respect to \( x \) since solving for \( y \) in \( x = 3y^2 + 18y \) is messy, i.e., \( y = \frac{-18 \pm \sqrt{324 + 144}}{6} \).

5. (2 points) Let \( R \) be the finite region bounded by the graph of \( y = 9 - e^{2x} \) and the \( x \)-axis on the interval \([\ln(3), \ln(5)]\). Revolve \( R \) around the horizontal line \( y = 10 \) to form a solid. By integrating with respect to \( x \), set up but do not evaluate a definite integral which represents the volume of the solid. Use proper notation.

\[
V = \int_{x_{\min}}^{x_{\max}} \left( \text{cross-sectional area of the slice at } x \right) \, dx
\]

\[
= \int_{\ln(3)}^{\ln(5)} \left( \pi \left( r_{\text{out}}^2 - r_{\text{in}}^2 \right) \right) \, dx
\]

\[
= \int_{\ln(3)}^{\ln(5)} \left( \pi (10 - (9 - e^{2x}))^2 - \pi (10 - 0)^2 \right) \, dx
\]

Solid shape: