MATH 221

Test 3

Fall 2017

Name ____________________________

NetID ___________________________

UIN ____________________________

Circle your TA discussion section.

- CD1, WF 9:00-10:50, Nigel Pynn-Coates
- CD2, WF 1:00-2:50, Chris Linden
- CD3, WF 9:00-10:50, Stefan Klajbor-Goderich
- CDA, WF 8:00-8:50, Hsin-Po Wang
- CDB, WF 9:00-9:50, Hsin-Po Wang
- CDC, WF 10:00-10:50, Albert Tamazyan
- CDD, WF 11:00-11:50, Dara Zirlin
- CDE, WF 12:00-12:50, Dara Zirlin
- CDF, WF 1:00-1:50, Albert Tamazyan
- CDH, WF 3:00-3:50, Xiaolong ‘Hans’ Han
- CDI, WF 8:00-8:50, Haojian Li
- CDJ, WF 9:00-9:50, Jianting ‘Jesse’ Huang
- CDK, WF 10:00-10:50, Haojian Li
- CDL, WF 11:00-11:50, Xiaolong ‘Hans’ Han
- CDM, WF 1:00-1:50, Dana Neidinger
- CDN, WF 2:00-2:50, Ningchuan Zhang
- CDO, WF 8:00-8:50, Lan Wang
- CDP, WF 9:00-9:50, Lan Wang
- CDQ, WF 10:00-10:50, Xinghua Gao
- CDR, WF 11:00-11:50, Xinghua Gao
- CDS, WF 12:00-12:50, Jianting ‘Jesse’ Huang
- CDT, WF 1:00-1:50, Ningchuan Zhang
- CDU, WF 2:00-2:50, Dana Neidinger

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quick working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.

FRONT OF ROOM – 314 Altgeld Hall
1. (12 points) At \( t \) hours, a population of bacteria is growing at a rate of \( \frac{21e^{\sqrt{t}}}{\sqrt{t}} \) bacteria per hour. Compute the change in population size between times \( t = 169 \) and \( t = 225 \). Simplify your answer.

\[
\text{(Net change in population from } t = 169 \text{ to } t = 225) = \int_{169}^{225} \left( \frac{21e^{\sqrt{t}}}{\sqrt{t}} \right) dt
\]

\[
= \int_{13}^{15} 21e^u \cdot 2du
\]

\[
= 42e^{15} - 42e^{13} \text{ bacteria}
\]

2. (12 points) Find the average value of the function \( f(x) = \frac{8x}{x^2 + 9} \) on the interval \([2, 11]\). Simplify your answer.

\[
f_{\text{ave}} = \frac{1}{11-2} \int_{2}^{11} \frac{8x}{x^2 + 9} \, dx
\]

\[
= \frac{1}{9} \int_{13}^{130} \frac{4}{u} \, du
\]

\[
= \frac{4}{9} \left[ \ln(u) \right]_{13}^{130}
\]

\[
= \frac{4}{9} \left[ \ln(130) - \ln(13) \right]
\]

\[
= \frac{4}{9} \ln \left( \frac{130}{13} \right)
\]

\[
= \frac{4}{9} \ln (10)
\]
3. (12 points) Evaluate the definite integral. Simplify your answer.

\[
\int_{-8}^{-6} \left( 15(x+12)^2 + \frac{(x+12)^9}{(x+12)^4 + 16} \right) \, dx
= \int_{-4}^{4} \left( 15u^2 + \frac{u^9}{u^{12} + 16} \right) \, du
= \int_{-4}^{4} 15u^2 \, du + \int_{-4}^{4} \frac{u^9}{u^{12} + 16} \, du
\]

\[
= 2 \int_{-4}^{4} 15u^2 \, du + 0
= 2 \cdot 5u^3 \bigg|_{0}^{4}
= 10 \cdot 4^3
= 640
\]

**Note 1:**
\[
F(u) = 15u^2
\]
\[
F(-u) = 15(-u)^2 = 15u^2 = F(u)
\]

*F is even*

**Note 2:**
\[
g(u) = \frac{u^9}{u^{12} + 16}
\]
\[
g(-u) = \frac{(-u)^9}{(-u)^{12} + 16} = \frac{-u^9}{u^{12} + 16} = -g(u)
\]

*g is odd*
4. (12 points) Evaluate the indefinite integral.

\[
\int 42e^{8x} (e^{42x} + 5)^{35} \, dx = \int e^{42x}(e^{42x} + 5)^{35} \cdot 42e^{42x} \, dx
\]

Let \( u = e^{42x} + 5 \)

\[
du = 42e^{42x} \, dx
\]

\[
u - 5 = e^{42x}
\]

\[
\int (u - 5)u^{35} \, du = \int (u^{36} - 5u^{35}) \, du
\]

\[
= \frac{1}{37} u^{37} - \frac{5}{36} u^{36} + C
\]

\[
= \frac{1}{37} (e^{42x} + 5)^{37} - \frac{5}{36} (e^{42x} + 5)^{36} + C
\]
5. (12 points) Evaluate the indefinite integral.

\[
\int \frac{42x^{13}}{x^{28} - 18x^{14} + 82} \, dx = \int \frac{3}{(x^{14})^2 - 18x^{14} + 82} \cdot 14x^{13} \, dx
\]

\[
\left(\begin{array}{c}
u = x^{14} \\
du = 14x^{13} \, dx
\end{array}\right) = \int \frac{3}{u^2 - 18u + 82} \, du
\]

\[
= \int \frac{3}{(u-9)^2 + 1} \, du
\]

\[
= \int \frac{3}{w^2 + 1} \, dw
\]

\[
= 3 \arctan(w) + C
\]

\[
= 3 \arctan(u-9) + C
\]

\[
= 3 \arctan(x^{14}-9) + C
\]
6. (10 points) Evaluate the indefinite integral.

\[
\int \frac{5x^2 - 43}{x + 3} \, dx = \left(5x - 15 + \frac{43}{x + 3}\right) \, dx
= \frac{5}{2} x^2 - 15x + \ln |x + 3| + C
\]

Method 2

\[
\int \frac{5x^2 - 43}{x + 3} \, dx = \int \frac{5(u^2 - 30u + 2)}{u} \, du
= \int \frac{5u^2 - 150u + 10}{u} \, du
= \int \frac{5u^2 - 150u + 214u}{u} \, du + C
= \frac{5}{3} (x + 3)^2 - 30(x + 3) + 214\ln (x + 3) + C
\]

7. (10 points) Evaluate the indefinite integral.

\[
\int 10x^9 \tan^3(x^{10}) \sec^{15}(x^{10}) \, dx = \int \tan^3(u) \sec^{15}(u) \, du
= \int \tan^2(u) \sec^{14}(u) \sec(u) \tan(u) \, du
= \int (\sec^2(u) - 1) \sec^{14}(u) \sec(u) \tan(u) \, du
= \int (w^2 - 1) w^{14} \, dw
= \int (w^{16} - w^{14}) \, dw
= \frac{1}{17} w^{17} - \frac{1}{15} w^{15} + C
= \frac{1}{17} \sec^{17}(u) - \frac{1}{15} \sec^{15}(u) + C
\]

\[
= \frac{1}{17} \sec^{17}(x^{10}) - \frac{1}{15} \sec^{15}(x^{10}) + C
\]
8. (5 points each) Let R be the finite region bounded by the graphs of \( y = 3 \sin x \), \( y = 6 \), \( x = 0 \), and \( x = \pi \). Set up, but do not evaluate, definite integrals which represent the following quantities. Integrate with respect to \( x \).

(a) The area of the region R.

\[
A = \int_{x_{\text{min}}}^{x_{\text{max}}} (\text{top} - \text{bottom}) \, dx
\]

\[
A = \int_{0}^{\pi} (6 - 3\sin(x)) \, dx
\]

(b) The volume of the solid with base R for which the cross-sections perpendicular to the \( x \)-axis are squares.

\[
V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{area} \, dx
\]

\[
V = \int_{0}^{\pi} (\text{side length})^2 \, dx
\]

\[
V = \int_{0}^{\pi} (6 - 3\sin(x))^2 \, dx
\]

(c) The volume of the solid formed when R is revolved around the horizontal line \( y = 8 \).

\[
V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{area} \, dx
\]

\[
V = \int_{0}^{\pi} (\pi \text{R}_{\text{out}}^2 - \pi \text{R}_{\text{in}}^2) \, dx
\]

\[
V = \int_{0}^{\pi} (\pi (8 - 3\sin(x))^2 - \pi (8 - 6)^2) \, dx
\]

(d) The volume of the solid formed when R is revolved around the vertical line \( x = -2 \).

\[
V = \int_{x_{\text{min}}}^{x_{\text{max}}} \text{area} \, dx
\]

\[
V = \int_{0}^{\pi} 2\pi r \, h \, dx
\]

\[
V = \int_{0}^{\pi} 2\pi (x - (-2))(6 - 3\sin(x)) \, dx
\]
Students – do not write on this page!

1. (12 points) ____________________

2. (12 points) ____________________

3. (12 points) ____________________

4. (12 points) ____________________

5. (12 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8a. (5 points) ____________________

8b. (5 points) ____________________

8c. (5 points) ____________________

8d. (5 points) ____________________

TOTAL (100 points) _____________