MATH 221 Test 2 Fall 2017

Name ____________________________ NetID ____________________________

UIN ____________________________

Circle your TA discussion section.

• CD1, WF 9:00-10:50, Nigel Pynn-Coates
• CD2, WF 1:00-2:50, Chris Linden
• CD3, WF 9:00-10:50, Stefan Klajbor-Goderich
• CDA, WF 8:00-8:50, Hsin-Po Wang
• CDB, WF 9:00-9:50, Hsin-Po Wang
• CDC, WF 10:00-10:50, Albert Tamazyan
• CDE, WF 12:00-12:50, Dara Zirlin
• CDF, WF 1:00-1:50, Albert Tamazyan
• CDH, WF 3:00-3:50, Xiaolong ‘Hans’ Han
• CDI, WF 8:00-8:50, Haojian Li
• CDJ, WF 9:00-9:50, Jianting ‘Jesse’ Huang

• CDK, WF 10:00-10:50, Haojian Li
• CDL, WF 11:00-11:50, Xiaolong ‘Hans’ Han
• CDM, WF 1:00-1:50, Dana Neidinger
• CDN, WF 11:00-11:50, Ningchuan Zhang
• CDO, WF 8:00-8:50, Lan Wang
• CD2, WF 1:00-1:50, Ningchuan Zhang
• CDQ, WF 10:00-10:50, Xinghua Gao
• CDR, WF 11:00-11:50, Xinghua Gao
• CDS, WF 12:00-12:50, Jianting ‘Jesse’ Huang
• CDT, WF 1:00-1:50, Ningchuan Zhang
• CDU, WF 2:00-2:50, Dana Neidinger

• Sit in your assigned seat (circled below).
• Do not open this test booklet until I say START.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• Remove hats and sunglasses.
• There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
• While the test is in progress, we will not answer questions concerning the test material.
• Do not leave early unless you are at the end of a row.
• Quit working and close this test booklet when I say STOP.
• Quickly turn in your test to me or a TA and show your Student ID.
1. (10 points) Determine the $x$-coordinate of the highest point on the graph of the following function.

$$f(x) = 1260 \arctan (3x) - 5 \ln (9x^2 + 1)$$

2. (10 points) A function $f(x)$ is differentiable everywhere and has the following second derivative.

$$f''(x) = \frac{(2x^2 - 288)(x + 3)^9(x^2 + 25)}{20e^{16-x}}$$

Find the intervals of concavity for $f(x)$ and state each $x$-value at which the graph of $f(x)$ has an inflection point.
3. (10 points) Let (0, 0) be the lower left corner and let (x, y) be the upper right corner of a rectangle as shown in the diagram. The upper right corner moves along the curve \( f(x) = 25e^{-3x} \) so that its x-coordinate is moving to the right at 4 cm/s. How quickly is the area of the rectangle changing at the moment that the upper right corner of the rectangle has an x-coordinate of 10 cm?
4. (10 points) Determine the formula for a function $f(x)$ which satisfies the following three conditions.

- $f''(x) = 800e^{4x} + 40\sin(x) - 25\cos(x)$
- $f'(0) = 80$
- $f(0) = 20$

5. (10 points) Express $10\ln(2) - 3\ln(10)$ as a single logarithm. Now use a linear approximation to estimate its value. Simplify and write your answer in decimal form.
6. (10 points) Fill in the missing information for the following theorems and tests.

**Mean Value Theorem**  Let $f$ be a function that satisfies the following two hypotheses.

1. $f$ is __________________________ on the closed interval $[a, b]$.

2. $f$ is __________________________ on the open interval $(a, b)$.

Then there is a number $c$ in $(a, b)$ such that __________________________.

**The First Derivative Test**  Suppose that $c$ is a critical number of a continuous function $f$.

- If $f'$ changes from positive to negative at $c$, then $f$ has a local __________________ at $c$.

- If $f'$ changes from negative to positive at $c$, then $f$ has a local __________________ at $c$.

**The Second Derivative Test**  Suppose $f''$ is continuous near $c$.

- If $f'(c) = 0$ and $f''(c) > 0$, then $f$ has a local __________________ at $c$.

- If $f'(c) = 0$ and $f''(c) < 0$, then $f$ has a local __________________ at $c$.

**Fundamental Theorem of Calculus, Part 2**

If $f$ is __________________________ on $[a, b]$, then $\int_a^b f(x) \, dx = __________________________$

where $F$ is any __________________________ of $f$. 
7. (10 points) Let \( g(x) = \int_{8}^{5x} \frac{1}{70} \left( \frac{t}{5} \right)^{10} \left( \frac{t}{5} \right)^{6} + 1 \, dt \). Evaluate \( \lim_{x \to \infty} \frac{x^3}{g(x)} \) given that \( \lim_{x \to \infty} g(x) = \infty \).

8. (10 points) Find the area of the region above the \( x \)-axis and below the curve \( y = \frac{4}{x^2} \) on the interval \([2, 5]\).
9. (10 points) Suppose that \( f \) is integrable on the interval \([3, 25]\). Given that \( \int_{3}^{25} f(x) \, dx = 60 \), \( \int_{3}^{9} f(x) \, dx = 25 \) and \( \int_{6}^{25} f(x) \, dx = 40 \), evaluate the following definite integrals.

(a) \( \int_{6}^{9} f(x) \, dx \)

(b) \( \int_{3}^{9} (5f(x) + 20) \, dx \)

(c) \( \int_{9}^{3} f(x) \, dx \)

(d) \( \int_{3}^{6} f(x) \, dx \)

(e) \( \int_{6}^{9} f(x) \, dx \)

10. (10 points) Evaluate the following definite integral and simplify your answer.

\[
\int_{15}^{40} \left(14 \sin (2x) - (7 \sin x + 2 \cos x)^2 + 45 \sin^2 x \right) \, dx
\]
Students – do not write on this page!

1. (10 points) ______________________
2. (10 points) ______________________
3. (10 points) ______________________
4. (10 points) ______________________
5. (10 points) ______________________
6. (10 points) ______________________
7. (10 points) ______________________
8. (10 points) ______________________
9. (10 points) ______________________
10. (10 points) _____________________

TOTAL (100 points) ________________