Circle your TA discussion section.

- CD1, WF 9:00-10:50, Nigel Pynn-Coates
- CD2, WF 1:00-2:50, Chris Linden
- CD3, WF 9:00-10:50, Stefan Klabjor-Goderich
- CDA, WF 8:00-8:50, Hsin-Po Wang
- CDB, WF 9:00-9:50, Hsin-Po Wang
- CDC, WF 10:00-10:50, Albert Tamazyan
- CDD, WF 11:00-11:50, Dara Zirlin
- CDE, WF 12:00-12:50, Dara Zirlin
- CDF, WF 1:00-1:50, Albert Tamazyan
- CDH, WF 3:00-3:50, Xiaolong 'Hans' Han
- CDI, WF 8:00-8:50, Haojian Li
- CDJ, WF 9:00-9:50, Jianting 'Jesse' Huang

- CDK, WF 10:00-10:50, Haojian Li
- CDL, WF 11:00-11:50, Xiaolong 'Hans' Han
- CDM, WF 1:00-1:50, Dana Neidinger
- CDN, WF 2:00-2:50, Ningchuan Zhang
- CDO, WF 8:00-8:50, Lan Wang
- CDP, WF 9:00-9:50, Lan Wang
- CDQ, WF 10:00-10:50, Xinghua Gao
- CDR, WF 11:00-11:50, Xinghua Gao
- CDS, WF 12:00-12:50, Jianting 'Jesse' Huang
- CDT, WF 1:00-1:50, Ningchuan Zhang
- CDU, WF 2:00-2:50, Dana Neidinger

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.
1. (10 points) Determine the $x$-coordinate of the highest point on the graph of the following function.

\[ f(x) = 1260 \arctan(3x) - 5 \ln(9x^2 + 1) \]

\[ f'(x) = 1260 \cdot \frac{1}{(3x)^2 + 1} \cdot 3 - 5 \cdot \frac{1}{9x^2 + 1} \cdot 18x \]

\[ = \frac{3780}{9x^2 + 1} - \frac{90x}{9x^2 + 1} \]

\[ = \frac{3780 - 90x}{9x^2 + 1} \]

\[ 90(42-x) \]

Values of $f'(x)$

\[ + + 0 - - \]

42

highest point at abs. max. when

\[ x = 42 \]

2. (10 points) A function $f(x)$ is differentiable everywhere and has the following second derivative.

\[ f''(x) = \frac{(2x^2 - 288)(x + 3)^2}{20e^{16-x}} (x + 25) \]

Find the intervals of concavity for $f(x)$ and state each $x$-value at which the graph of $f(x)$ has an inflection point.

\[ f''(x) = \frac{2(x^2-144)(x+3)^9(x + 25)}{20e^{16-x}} \]

\[ = \frac{2(x+12)(x-12)(x+3)^9(x + 25)}{20e^{16-x}} \]

Values of $f''(x)$

\[ - 0 + 0 - 0 + \]

12

-12 -3

f conc. down $(-\infty, -12)$
f conc. up $(-12, -3)$
f conc. down $(-3, 12)$
f conc. up $(12, \infty)$
inf. pts at $x = -12, -3, 12$
3. (10 points) Let (0, 0) be the lower left corner and let \((x, y)\) be the upper right corner of a rectangle as shown in the diagram. The upper right corner moves along the curve \(f(x) = 25e^{-3x}\) so that its \(x\)-coordinate is moving to the right at 4 cm/s. How quickly is the area of the rectangle changing at the moment that the upper right corner of the rectangle has an \(x\)-coordinate of 10 cm?

Given: \(\frac{dx}{dt} = 4 \text{ cm/s}\)

Want: \(\frac{dA}{dt}\) \(|\) \(x = 10 \text{ cm}\)

\[
\text{Area} = \text{Width} \cdot \text{Height} \\
A = x \cdot y \\
A = x \cdot 25e^{-3x} = 25xe^{-3x}
\]

\[
\frac{d}{dt}(A) = \frac{d}{dt}(25xe^{-3x})
\]

\[
\frac{dA}{dt} = 25 \frac{dx}{dt} e^{-3x} + 25x \cdot (-3e^{-3x} \frac{dx}{dt})
\]

\[
\left. \frac{dA}{dt} \right|_{x=10 \text{ cm}} = 25 \left( \frac{4}{1} e^{-30} + 25(10)(-3e^{-30}) \left( \frac{4}{1} \right) \right)
\]

\[
\left. \frac{dA}{dt} \right|_{x=10 \text{ cm}} = 100e^{-30} - 3000e^{-30}
\]

\[
\frac{dA}{dt} \left|_{x=10 \text{ cm}} = \frac{-2900e^{-30} \text{ cm}^2}{5} \right.
\]
4. (10 points) Determine the formula for a function $f(x)$ which satisfies the following three conditions.

- $f''(x) = 800e^{4x} + 40 \sin (x) - 25 \cos (x)$
- $f'(0) = 80$
- $f(0) = 20$

\[ f'(x) = 200e^{4x} - 40\cos(x) - 25\sin(x) + C_1 \]
\[ f'(0) = 80 \Rightarrow 200e^{4\cdot0} - 40\cos(0) - 25\sin(0) + C_1 = 80 \]
\[ 80 - 40 - 0 + C_1 = 80 \]
\[ C_1 = -80 \]

\[ f'(x) = 200e^{4x} - 40\cos(x) - 25\sin(x) - 80 \]

\[ f(x) = 50e^{4x} - 40\sin(x) + 25\cos(x) - 80x + C_2 \]

\[ f(0) = 20 \Rightarrow 50 - 0 + 25 - 0 + C_2 = 20 \Rightarrow C_2 = -55 \]

\[ f(x) = 50e^{4x} - 40\sin(x) + 25\cos(x) - 80x - 55 \]

5. (10 points) Express $10\ln(2) - 3\ln(10)$ as a single logarithm. Now use a linear approximation to estimate its value. Simplify and write your answer in decimal form.

\[ 10\ln(2) - 3\ln(10) = \ln(2^{10}) - \ln(10^3) \]
\[ = \ln\left(\frac{2^{10}}{10^3}\right) \]
\[ = \ln\left(\frac{1024}{1000}\right) \]

we will find the line tangent to $f(x) = \ln(x)$ at $x = 1$ (note: $f'(x) = \frac{1}{x}$)

point: $(1, f(1)) = (1, \ln(1)) = (1, 0)$

slope: $f'(1) = \frac{1}{1} = 1$

tangent line: $y - 0 = 1 \cdot (x - 1)$

\[ y = x - 1 \]

\[ \ln(x) \approx x - 1 \]

for $x$ near 1

\[ \ln\left(\frac{1024}{1000}\right) \approx \frac{1024}{1000} - 1 \]

\[ \ln\left(\frac{1024}{1000}\right) \approx \frac{24}{1000} \]

\[ 10\ln(2) - 3\ln(10) = \ln\left(\frac{1024}{1000}\right) \approx 0.024 \]
6. (10 points) Fill in the missing information for the following theorems and tests.

**Mean Value Theorem** Let $f$ be a function that satisfies the following two hypotheses.

(1) $f$ is continuous on the closed interval $[a, b]$.

(2) $f$ is differentiable on the open interval $(a, b)$.

Then there is a number $c$ in $(a, b)$ such that 

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$ 

**The First Derivative Test** Suppose that $c$ is a critical number of a continuous function $f$.

- If $f'$ changes from positive to negative at $c$, then $f$ has a local **maximum** at $c$.

- If $f'$ changes from negative to positive at $c$, then $f$ has a local **minimum** at $c$.

**The Second Derivative Test** Suppose $f''$ is continuous near $c$.

- If $f'(c) = 0$ and $f''(c) > 0$, then $f$ has a local **minimum** at $c$.

- If $f'(c) = 0$ and $f''(c) < 0$, then $f$ has a local **maximum** at $c$.

**Fundamental Theorem of Calculus, Part 2**

If $f$ is continuous on $[a, b]$, then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

where $F$ is any **antiderivative** of $f$. 
7. (10 points) Let $g(x) = \int_{\frac{1}{5}}^{\frac{5}{3}} \frac{1}{t^3} (\frac{1}{t})^{10} \, dt$. Evaluate $\lim_{x \to \infty} \frac{x^3}{g(x)}$ given that $\lim_{x \to \infty} g(x) = \infty$.

From Fundamental Theorem of Calculus (part 1) and the chain rule, $g'(x) = \frac{\frac{5x}{3}}{(\frac{5x}{3})^8 + 1}$.

$\lim_{x \to \infty} \frac{x^3}{g(x)} = \lim_{x \to \infty} \frac{3x^2}{g'(x)} = \lim_{x \to \infty} \frac{3x^2}{\frac{14x^{10}}{x^8 + 1}} = \lim_{x \to \infty} \frac{3x^2}{\frac{14x^{10} + 3x^2}{x^{10}}}.$

$= \lim_{x \to \infty} \frac{3x^2}{\frac{14x^{10}}{x^8 + 1}} = \lim_{x \to \infty} \frac{3x^{10} + 3x^2}{14x^{10}}.$

$= \lim_{x \to \infty} \left( \frac{3x^{10}}{14x^{10}} + \frac{3x^2}{14x^{10}} \right) = \lim_{x \to \infty} \left( \frac{42 + 42}{x^8} \right) = \frac{42}{1} = 42.$

8. (10 points) Find the area of the region above the x-axis and below the curve $y = \frac{4}{x^3}$ on the interval $[2, 5]$.

Area = $\int_{2}^{5} \frac{4}{x^2} \, dx = \int_{2}^{5} 4x^{-2} \, dx$.

$= \left. -4x^{-1} \right|_{2}^{5}$.

$= \frac{4}{5} - \frac{4}{2}$.

$= \frac{4}{5} - (\frac{4}{2})$.

$= \frac{4}{5} - 2$.

$= \frac{6}{5} = 1.2.$
9. (10 points) Suppose that $f$ is integrable on the interval $[3, 25]$. Given that $\int_{3}^{25} f(x) \, dx = 60$, $\int_{3}^{9} f(x) \, dx = 25$ and $\int_{6}^{25} f(x) \, dx = 40$, evaluate the following definite integrals.

(a) $\int_{6}^{9} f(x) \, dx = 0$

(b) $\int_{3}^{9} (5f(x) + 20) \, dx = 5 \int_{3}^{9} f(x) \, dx + \int_{3}^{9} 20 \, dx$

(c) $\int_{3}^{9} f(x) \, dx = \int_{3}^{9} f(x) \, dx - \int_{3}^{9} f(x) \, dx = 125 + 120 = 245$

(d) $\int_{3}^{6} f(x) \, dx = \int_{3}^{25} f(x) \, dx + \int_{25}^{6} f(x) \, dx$

(e) $\int_{6}^{9} f(x) \, dx = \int_{6}^{25} f(x) \, dx - \int_{25}^{6} f(x) \, dx + \int_{6}^{25} f(x) \, dx = 40 - 60 + 25 = 0$

10. (10 points) Evaluate the following definite integral and simplify your answer.

$\int_{15}^{40} (14 \sin (2x) - (7 \sin x + 2 \cos x)^2 + 45 \sin^2 x) \, dx =$

$\int_{15}^{40} (4,25 \sin^2 x \cos(x) - (49 \sin^2 x) + 28 \sin(x) \cos(x) + 4 \cos^2(x) + 45 \sin^2(x) \cos(x)) \, dx$

$= \int_{15}^{40} (28 \sin(x) \cos(x) - 49 \sin^2(x) - 28 \sin(x) \cos(x) - 4 \cos^2(x) + 45 \sin^2(x)) \, dx$

$= \int_{15}^{40} (-4 \sin^2(x) - 4 \cos^2(x)) \, dx$

$= \int_{15}^{40} -4 \, dx = -4x \bigg|_{15}^{40} = -4(40) - (-4(15))$

$= -100$
Students – do not write on this page!

1. (10 points) ____________________

2. (10 points) ____________________

3. (10 points) ____________________

4. (10 points) ____________________

5. (10 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8. (10 points) ____________________

9. (10 points) ____________________

10. (10 points) ____________________

TOTAL (100 points) ________________