Math 221 CD1: Worksheet 7
September 20, 2017

1. Find the derivative $f'(x)$ of each of the following functions.
   (a) $f(x) = \sin(\sqrt{\cos x})$
   (b) $f(x) = \left(\frac{(x^7 - 4)^5}{x^7 + 1}\right)^3$
   (c) $f(x) = 3x^2 + 2x + e^{3x^2}$
   (d) $f(x) = \arctan(x^2 + 7x^2)$

2. Suppose that $g(0) = 0$ and that $g'(0) = 2$. What is the derivative of $g(g(g(x)))$ at $x = 0$?

3. (a) The volume of a spherical balloon, as a function of the radius $r$, is $V = \frac{4}{3} \pi r^3$. Compute $\frac{dV}{dr}$. What is $\frac{dV}{dr}$ measuring? What are the units of $\frac{dV}{dr}$?
   (b) Suppose you pump air into the balloon so that the radius, as a function of time $t$, is given by $r = \sqrt{t} + 1$. Compute $\frac{dr}{dt}$. What is $\frac{dr}{dt}$ measuring? What are the units of $\frac{dr}{dt}$?
   (c) Compute $\frac{dV}{dt}$. What is $\frac{dV}{dt}$ measuring? What are the units of $\frac{dV}{dt}$?

4. Find $\frac{dy}{dx}$ for each of the following equations.
   (a) $x^3 + y^4 = 7$
   (b) $y = \sin(3x + 4y)$
   (c) $y = x^2y^3 + x^3y^2$
   (d) $e^{xy} = e^{4x} - e^{5y}$
   (e) $\cos^2 x + \cos^2 y = \cos(2x + 2y)$

5. Find an equation of the line tangent to the graph of $x^2 + (y - x)^3 = 9$ at $x = 1$.

6. Find $\frac{dy}{dx}$ for the following functions.
   (a) $y = xe^x$
   (b) $y = (3x^2 + 5)^{1/x}$
   (c) $y = (\sin x)^{x^2}$
   (d) $y = \sqrt{x^2} e^{x^2}$
   (e) $y = x^{\ln x}(\sec x)^{3x}$
7. Given that \( f'(2) = 2, \ f'(3) = -1, \ g(1) = 2, \ g'(1) = -2, \ g(0) = 3, \) and \( g'(0) = 8, \) calculate \( h'(1) \) and \( h'(0), \) where \( h(x) = f(g(x)). \)

8. A function is called odd if \( f(-x) = -f(x) \) for all \( x \) and even if \( f(-x) = f(x) \) for all \( x. \)
   (a) Sketch a graph of an odd function; sketch a graph of an even function.
   (b) Show that the derivative of an odd function is even and that the derivative of an even function is odd.
   (c) Show that if \( f(x) \) is odd, then \( f(0) = 0. \) Is the same true for even function?
   (d) Give an example of a function that is both odd and even. Prove that your example is the only function that is both odd and even.

9. Find all points \((x, y)\) on the curve \( x^{2/3} + y^{2/3} = 8 \) where the lines tangent to the curve at \((x, y)\) have slope \(-1.\)

10. Consider the function \( f(x) = \frac{x^5 e^x (4x + 3)}{5 \ln x (3 - x)^2}. \) Find an equation of the line tangent to the graph of \( f \) at \( x = 1.\)

11. Find \( f'(x), \) where \( f(x) = 3 \sqrt{(1 - x^2)^6 + \frac{\sin^8 x + 1}{\cos x}}. \)

12. (a) Does the function assigning to each UIUC student their UIN have an inverse?
    (b) Does the function assigning to each person their birth date have an inverse?
    (c) Does \( \ln(x^2) \) have an inverse function? What about \( \ln(2x)? \) If the answers differ, what is different about these two functions?