Math 221 CD1: Worksheet 23
November 15, 2017

1. Use \( \int_{0}^{\pi/2} \sin(x) \, dx \) to compute \( \int_{0}^{1} \arcsin(x) \, dx \).
   (Hint: interpret these integrals as areas and be sure to sketch pictures.)

2. Suppose that \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\). Find \( \int_{a}^{b} 2f(x)f'(x) \, dx \).

3. Sketch the region enclosed by the given curves and find its area twice, once using an integral with respect to \( x \) and once using an integral with respect to \( y \).
   (a) \( y = 3, \ x = 0, \) and \( y = x^3 \)
   (b) \( y = x + 5, \ y = x^2 - 1 \)

4. Sketch the region enclosed by the curves \( y = \sin x \) and \( y = \cos x \) on \( \pi/4 \leq x \leq 5\pi/4 \) and find its area.

5. Sketch the region enclosed by the curves \( y = x^2 \) and \( y = 4x - x^2 \) and find its area.

6. Sketch the region enclosed by the curves \( y = \sinh x, \ y = e^{-x}, \ x = 0, \) and \( x = 2, \) and find its area.

7. Find the number \( b \) such that the line \( y = b \) divides the region bounded by the curves \( y = x^2 \) and \( y = 4 \) into two regions with equal areas.