Math 221 CD1: Worksheet 1
August 30, 2017

1. In your group, think of four functions, including a polynomial, a rational function that is not a polynomial, an exponential function, and a trigonometric function. First make sure you know what these terms mean. For each function, try and sketch a graph. What can you say about inverses of these functions? Do you remember any identities involving the trigonometric function you thought of?

2. For each of the following, find the equation of the line, the $y$-intercept, $x$-intercept, slope, and whether the line is increasing or decreasing. Then sketch the line.
   (a) The line with slope $-2$ and $y$-intercept 1.
   (b) The line passing through the point $(1, -1)$ with slope 3.
   (c) The line through the points $(1, 2)$ and $(3, 8)$. What is the distance between these two points?

3. For each pair of lines from the previous problem, find any points of intersection. Sketch all three lines on the same coordinate plane.

4. Given that $f(1) = 1$ and the $y$-intercept of the tangent line is at 3, what is $f'(1)$? Sketch the graph of a nonlinear function satisfying this condition.

5. City B is 100 kilometres NE of City A. If you travel at 50 kilometres per hour, how long will it take you to get from City A to City B? What is the slope of the line connecting City A and City B? If we pick a coordinate system so that City A is at the origin, what are the coordinates of City B? What changes if City B is ENE of City A instead of NE? Compute just the $y$-coordinate in this case.

6. You are at a sports game and someone is firing a T-shirt cannon. Your friendly neighbourhood mathematician has determined that, after $t$ seconds, the height (in feet) and the velocity (in feet per second) of the shirt are given by
   
   \[ h(t) = -5t^2 + 50t + 6, \quad v(t) = h'(t) = -10t + 50. \]

   (a) At what speed was the shirt fired out of the cannon?
   (b) From what height was it fired?
   (c) When does the shirt reach its highest point?
   (d) What is the highest point it reaches?
   (e) The cannon is fired in your direction. What additional information would you need to determine if you can catch the shirt?
   (f) You are sitting 70 feet away and 111 feet up and the shirt comes right to you. Can you now determine the missing information you wanted in part (e)?
7. Find the following limits, if they exist:

(a) \( \lim_{x \to 1} e^{2x} + 1 \) and \( \lim_{x \to \ln(3)} e^{2x} + 1 \);

(b) \( \lim_{x \to -1^-} \frac{x^2}{x+1} \) and \( \lim_{x \to -1^+} \frac{x^2}{x+1} \); \( \lim_{x \to -1} \frac{x^2}{x+1} \);

(c) \( \lim_{x \to -1^-} \frac{x}{x^2 + 1} \) and \( \lim_{x \to -1^+} \frac{x}{x^2 + 1} \); \( \lim_{x \to -1} \frac{x}{x^2 + 1} \);

(d) \( \lim_{x \to 0} \ln(x^2) \) and \( \lim_{x \to 0} 2 \ln(x) \);

(e) \( \lim_{x \to \frac{\pi}{2}} \tan(x) \);

(f) \( \lim_{x \to \infty} \arctan(x) \).

8. Sketch the graph of a function \( f(x) \) satisfying the following conditions:

(a) \( \lim_{x \to -1} f(x) = 3 \);

(b) \( f(-1) = 5 \);

(c) \( \lim_{x \to 1^-} f(x) = -1 \);

(d) \( \lim_{x \to 1^+} f(x) = 2 \);

(e) has a vertical asymptote at \( x = 3 \).