Name:

Be sure to show all work and state all tests that you use to receive full credit.

1. Determine the convergence of the series \( \sum_{n=1}^{\infty} \frac{n^2 - 1}{n^{7/2} + n + 2} \).

   **Solution 1:** We use the comparison test. Note that \( \frac{n^2 - 1}{n^{7/2} + n + 2} \leq \frac{n^2}{n^{7/2} + n + 2} \leq \frac{n^2}{n^{7/2}} = \frac{1}{n^{3/2}} \).

   We know that \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \) converges by the \( p \)-test, so thus the series \( \sum_{n=1}^{\infty} \frac{n^2 - 1}{n^{7/2} + n + 2} \) also converges.

   **Solution 2:** We use the limit comparison test. We will compare the series to \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \), which we know converges by the \( p \)-test.

   \[
   \lim_{n \to \infty} \frac{n^2 - 1}{n^{7/2} + n + 2} = \lim_{n \to \infty} \frac{n^{7/2} - n^{3/2}}{n^{7/2}} = 0
   \]

   It is easy to show that the limit is 1, so the series converges by the limit comparison test.

2. Use the ratio test to determine the convergence of the series \( \sum_{n=1}^{\infty} \frac{2^n}{n!} \). **Solution:** To use the ratio test we take the following limit.

   \[
   \lim_{n \to \infty} \left| \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} \right| = \lim_{n \to \infty} \left| \frac{2}{n + 1} \right| = 0
   \]

   Since the limit is less than 1, the series converges by the ratio test.
3. Determine if the series \( \sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n} \) is divergent, absolutely convergent, or conditionally convergent.

**Solution:** If we check the \( n \)th term test, the terms limit to \( \ln 1 = 0 \), so the test is inconclusive. We then check for absolute convergence. When we examine the series \( \sum_{n=1}^{\infty} \frac{\ln n}{x} \) and compare it to \( \sum_{n=1}^{\infty} \frac{1}{n} \), the comparison test shows us that the series diverges. Thus the series is not absolutely convergent. It remains to check for conditional convergence. We already know that \( \lim_{n \to \infty} \frac{\ln n}{n} = 0 \), and as the terms are decreasing, the alternating series test shows that the series is convergent. Thus the series is conditionally convergent.