Quiz 8 Solutions

Math 231

March 30, 2007

1. Determine if the series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ converges. If it does, evaluate the sum.

Solution: Use the fact that this is a telescoping series. $\ln(n/(n+1)) = \ln n - \ln(n+1)$, so the $k$th partial sum is given by

$$S_k = \sum_{n=1}^{k} \ln\left(\frac{n}{n+1}\right)$$

$$= \sum_{n=1}^{k} \ln n - \ln(n+1)$$

$$= (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + \ldots + (\ln k - \ln(k+1))$$

$$= \ln 1 - \ln(k+1)$$

$$= -\ln(k+1)$$

We know that a series converges if and only if the sequence of partial sums converges, so taking the limit we get

$$\lim_{k \to \infty} S_k = \lim_{n \to \infty} -\ln(k+1) = -\infty.$$

Thus the series diverges.

2. Consider the series $\sum_{n=1}^{\infty} \frac{3}{2^n}$.

(a) What type of series is this?

Solution: This is a geometric series.

(b) Determine if the series converges. If it converges, evaluate the sum.

Solution: This is a geometric series with $r = 1/2$. Since $r < 1$ we know that the series converges, and we can evaluate the sum:

$$\sum_{n=1}^{\infty} \frac{3}{2^n} = \frac{3/2}{1 - 1/2} = \frac{3/2}{1/2} = 3.$$
3. Use the integral test to determine if the series \( \sum_{n=1}^{\infty} \frac{4}{(n + 3)^{3/2}} \) converges.

**Solution:** We know that the series \( \sum_{n=1}^{\infty} \frac{4}{(n + 3)^{3/2}} \) converges if and only if the integral \( \int_{1}^{\infty} \frac{4}{(x + 3)^{3/2}} \, dx \) converges. Thus we compute the integral.

\[
\int_{1}^{\infty} \frac{4}{(x + 3)^{3/2}} \, dx = \left[ -\frac{8}{(x + 3)^{1/2}} \right]_{1}^{\infty}
= \lim_{a \to \infty} \left( -\frac{8}{(a + 3)^{1/2}} - \frac{8}{(1 + 3)^{1/2}} \right)
= 4
\]

Since the integral converges we know that the series must also converge.