Exam 3

Math 231

November 29, 2006

Name:

Directions

1. Be sure to show all work to receive full credit.

2. No calculators, books, notes or cell phones are allowed during the exam.

3. To receive full credit, you must state all rules that you use.
1. (a) (3 pts) Plot the point \((-1, \frac{3\pi}{4})\) in polar coordinates and find its rectangular coordinates.

(b) (3 pts) Draw the graph of the curve \(r = 3\) and give the equation of the curve in rectangular coordinates.

(c) (3 pts) Express the equation \(x = 3y\) in polar coordinates.
2. (a) (3 pts) Sketch the graph of \( r = 3 + 6 \sin \theta \).

(b) (9 pts) Find the area of the region bounded by the inner loop of the limacon \( r = 3 + 6 \sin \theta \).
3. (4 pts) Set up but do not compute the integral for the area inside $r = 2 + \cos \theta$ and outside $r = 2$. 
4. (a) (3 pts) Sketch the curve \( x = 2 \sin \theta, \ y = 2 \cos \theta. \)

(b) (6 pts) Find the equation of the line tangent to the curve at \((\sqrt{2}, \sqrt{2})\).
5. (a) (4 pts) Set up but do not compute the integral that gives the positive area between the curve 
\( x = t^2, \ y = \cos t; \ -\pi/2 \leq t \leq 0 \) and the x-axis.

(b) (7 pts) Find the volume of the solid generated by revolving the curve \( x = 5t + 2, \ y = t^3; \ -1 \leq t \leq 1 \) about the y-axis.
6. (a) (4 pts) Set up but do not compute the integral for the length of the curve \( y = x^{4/3}, \ -1 \leq x \leq 1 \).

(b) (4 pts) Set up but do not compute the integral for the surface area of the solid generated by revolving the curve \( y = \sqrt{x}, \ 1 \leq x \leq 4 \) about the \( x \)-axis.
7. (a) (7 pts) Find the arc length of the curve \( x = \sin t - \cos t, \ y = \sin t + \cos t; \ \frac{\pi}{4} \leq t \leq \frac{\pi}{2}. \)

(b) (7 pts) Find the surface area of the surface generated by revolving \( x = 2t + 1, \ y = t^2 + t; \ 0 \leq t \leq 3 \) about the \( y \)-axis.
8. (a) (3 pts) Sketch the conic section \( \frac{x^2}{9} + \frac{y^2}{16} = 1 \) and identify its type.

(b) (4 pts) Find the focus and directrix of the parabola \( \frac{1}{16} y^2 = x \).
9. (6 pts) Solve the initial value problem \( \frac{dy}{dx} = \frac{7}{y} \); \( y(0) = 6 \).