MATH 124 SPRING 2009

EXAM 1

Name: Solutions

Section:_______

No Calculators Allowed. Show all work.

Problem 1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6, 8\}$ and $C = \{1, 3, 5, 7, 9\}$. Write out the following sets using roster notation.

(a) (2 pts.) $(A \cup B^c)^c$

$$\{4, 5, 6, 8, 3\}$$

(b) (2 pts.) $B \cap (A^c \cup C)$

$$\{3, 4, 5, 6, 8, 3\}$$

(c) (2 pts.) $(A \cap C) \cap (A \cap C^c)$

$$\emptyset$$

Problem 2. (5 pts.) If $S = \{a, b, c, d\}$ and $P(\{a\}) = 2P(\{b\})$, $P(\{c\}) = \frac{2}{15}$, and $P(\{d\}) = \frac{1}{15}$, what is $P(\{a\})$?

$$P(a) + P(b) + P(c) + P(d) = 1$$

$$P(a) + \frac{1}{2}P(a) + \frac{2}{15} + \frac{1}{15} = 1$$

$$\frac{3}{2}P(a) = \frac{12}{15}$$

$$P(a) = \frac{12}{15} = \frac{8}{15}$$
Problem 3. Let $U$ be the set of all students at U of I and let

$C = \{x \mid x \text{ lives on-campus}\}$

$U = \{x \mid x \text{ lives in Urbana}\}$

$P = \{x \mid x \text{ owns a pet}\}$

$V = \{x \mid x \text{ owns a vehicle}\}$

Write the set that represents each of the following:

(a) (2 pts.) Students at U of I who live in Urbana and own a pet but not a vehicle.

\[ U \cap P \cap V^c \]

(b) (2 pts.) Students at U of I who live off-campus and own either a pet or a vehicle.

\[ C^c \cap (P \cup V) \]

(c) (2 pts.) Students at U of I who do live on-campus but not in Urbana or own both a pet and a vehicle.

\[ (C \cap U^c) \cup (P \cap V) \]
Problem 4. A survey of 300 households found that 125 owned VCRs, 225 owned DVD players and 75 owned both.

(a) (2 pts.) Draw a Venn diagram representing this situation.

(b) (2 pts.) How many households owned a VCR but not a DVD player?

50

(c) (2 pts.) How many households owned either a VCR or a DVD player?

275

Problem 5. (4 pts.) Two 6 sided dice are rolled. List the outcomes in the event that the sum is 9 or 3.

\[ \{(3,6), (4,5), (5,4), (6,3), (1,2), (2,1)\} \]
Problem 6. Two standard 6-sided dice are rolled.

(a) (3 pts.) What is the probability that the sum is equal to 4?

\[ \frac{3}{36} = \frac{1}{12} \]

(b) (3 pts.) What is the probability that both dice show an odd number?

\[ \frac{9}{36} = \frac{1}{4} \]

Problem 7. (4 pts.) If the odds of the Chicago Cubs winning the World Series are 1:5, what is the probability that they will win the World Series?

\[ \frac{1}{1+5} = \frac{1}{6} \]
Problem 8. A factory has two machines that produce all of their product. The first machine produces 60 percent of the total product, while the second machine produces 40 percent of the total product. The first machine produces a defective product 2 percent of the time; whereas the second machine produces a defective product 3 percent of the time.

(a) (3 pts.) Draw the corresponding probability tree.

(b) (2 pts.) Find the probability that a given product is defective.

\[ P(D) = (0.60)(0.02) + (0.40)(0.03) \]

(c) (2 pts.) Given that the product is produced by the second machine, find the probability that the product is defective.

\[ P(D|M_2) = 0.3 \]

(d) (3 pts.) Given that a defective product is produced, what is the probability that it was produced by the second machine?

\[ P(M_2 | D) = \frac{(0.40)(0.03)}{(0.60)(0.02) + (0.40)(0.03)} \]
Problem 9. (6 pts.) A picture is to be taken of eleven athletes. Four of eleven football players will line up in order on the left; three of nine basketball players will line up in order in the center; and four of seven golfers will line up in order on the right. In how many ways is this possible?

\[ \frac{P(11,4)}{7!} \times \frac{P(9,3)}{6!} \times \frac{P(7,4)}{3!} \]

Problem 10. A two-card hand is dealt from a standard deck of 52 cards.

(a) (4 pts.) What is the probability of getting two kings?

\[ \frac{C(4,2)}{C(52,2)} = \frac{4!}{2! \cdot 2!} \cdot \frac{52!}{38 \cdot 2!} = \frac{4! \cdot 50!}{2! \cdot 52!} \]

(b) (4 pts.) What is the probability of getting two cards in the same suit?

\[ \frac{C(4,1) \cdot C(13,2)}{C(52,2)} = \frac{4! \cdot 13!}{11 \cdot 2!} \cdot \frac{52!}{38 \cdot 2!} = \frac{4! \cdot 13! \cdot 50!}{11 \cdot 52!} \]
Problem 11. (7 pts.) A Bernoulli trial with \( p = 0.4 \) is repeated twelve times. What is the probability of at most two successes?

\[
P(k \leq 2) = \binom{12}{2} p^2 q^{10} + \binom{12}{1} p^1 q^{11} + \binom{12}{0} p^0 q^{12}
\]

\[
= \frac{12!}{10!2!} (0.4)^2 (0.6)^{10} + \frac{12!}{11!1!} (0.4)^1 (0.6)^{11}
\]

\[
+ \frac{12!}{12!0!} (0.4)^0 (0.6)^{12}
\]

Problem 12. A pair of fair dice are rolled. Consider the events

\[ E: \text{ The sum is 8} \]
\[ F: \text{ At least 1 of the dice is a 3} \]

(a) (2 pts.) Compute \( P(E|F) \)

\[
P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2}{36} \]

(b) (2 pts.) Compute \( P(F|E) \)

\[
P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{2}{36} \cdot \frac{36}{5} = \frac{2}{5}
\]

(c) (4 pts.) Are \( E \) and \( F \) independent? You must mathematically justify your answer.

No. \( P(E) \cdot P(F) = \frac{5}{36} \cdot \frac{11}{36} = \frac{55}{(36)^2} \)

\[
P(E \cap F) = \frac{2}{36} = \frac{72}{(36)^2}
\]

\( P(E) \cdot P(F) \neq P(E \cap F) \)
Problem 13. Evaluate the following. Your answer must be a number.

(a) (2 pts.) \( 5! \)
\[
5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120
\]

(b) (2 pts.) \( P(10,3) \)
\[
P(10,3) = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720
\]

(c) (2 pts.) \( C(8,5) \)
\[
C(8,5) = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56
\]

Problem 14. (6 pts.) How many distinct words can be spelled using all the letters in the word ILLINOIS? (Express your answer in terms of factorials.)

\[
\frac{8!}{3! \cdot 2!}
\]
Problem 15. (6 pts.) A box has four defective iPods and six non-defective ones. If two are picked at random, what is the probability that both are defective?

\[
\frac{\binom{4}{2}}{\binom{10}{2}} = \frac{\frac{4!}{2!2!}}{\frac{10!}{8!2!}} = \frac{4! \cdot 8!}{2! \cdot 10!}
\]

Problem 16. An sack contains 7 balls, each of a different color. Four balls are selected sequentially and the color is noted. In how many ways can this be done:

(a) (3 pts.) With replacement?

\[7 \cdot 7 \cdot 7 \cdot 7 = 7^4\]

(b) (3 pts.) Without replacement?

\[P(7, 4) = \frac{7^4}{4!} = 7 \cdot 6 \cdot 5 \cdot 4\]