Statistics 20: Quiz 2 Solutions
Summer Session 2007

1. Determine whether each of the following samples is a simple random sample (SRS). Give a brief explanation of why the sample is or is not an SRS.

   (a) [2 points] A deck of 36 cards is shuffled. Then a card is chosen at random from the deck. The value of the card is recorded, and the card is returned to the deck. Repeat the procedure six times to get a sample of size six.

   Solution: This is **NOT** a simple random sample because the same card can be chosen more than once. A simple random sample of size 6 must have 6 distinct cards.

   (b) [2 points] A deck of 36 cards is shuffled. To get a sample of size six, randomly draw an integer i from 1 to 6, and draw every sixth card beginning with the ith card from the top to the bottom of the deck.

   Solution: This **IS** a simple random sample. Every possible sample of size six has the same chance to be chosen.

   (c) [2 points] The cards in a deck are numbered $1, 2, ...36$. Randomly draw an integer i from 1 to 31, and choose cards $i, i+1, .., i+5$.

   Solution: This is **NOT** a simple random sample because every possible sample of size six does not have a chance to be chosen. Only cards that are in sequential order can be chosen.

2. Urn A has four red, three blue, and two green balls. Urn B has two red, three blue, and four green balls. A ball is drawn from urn A and put into urn B, and then a ball is drawn from urn B.

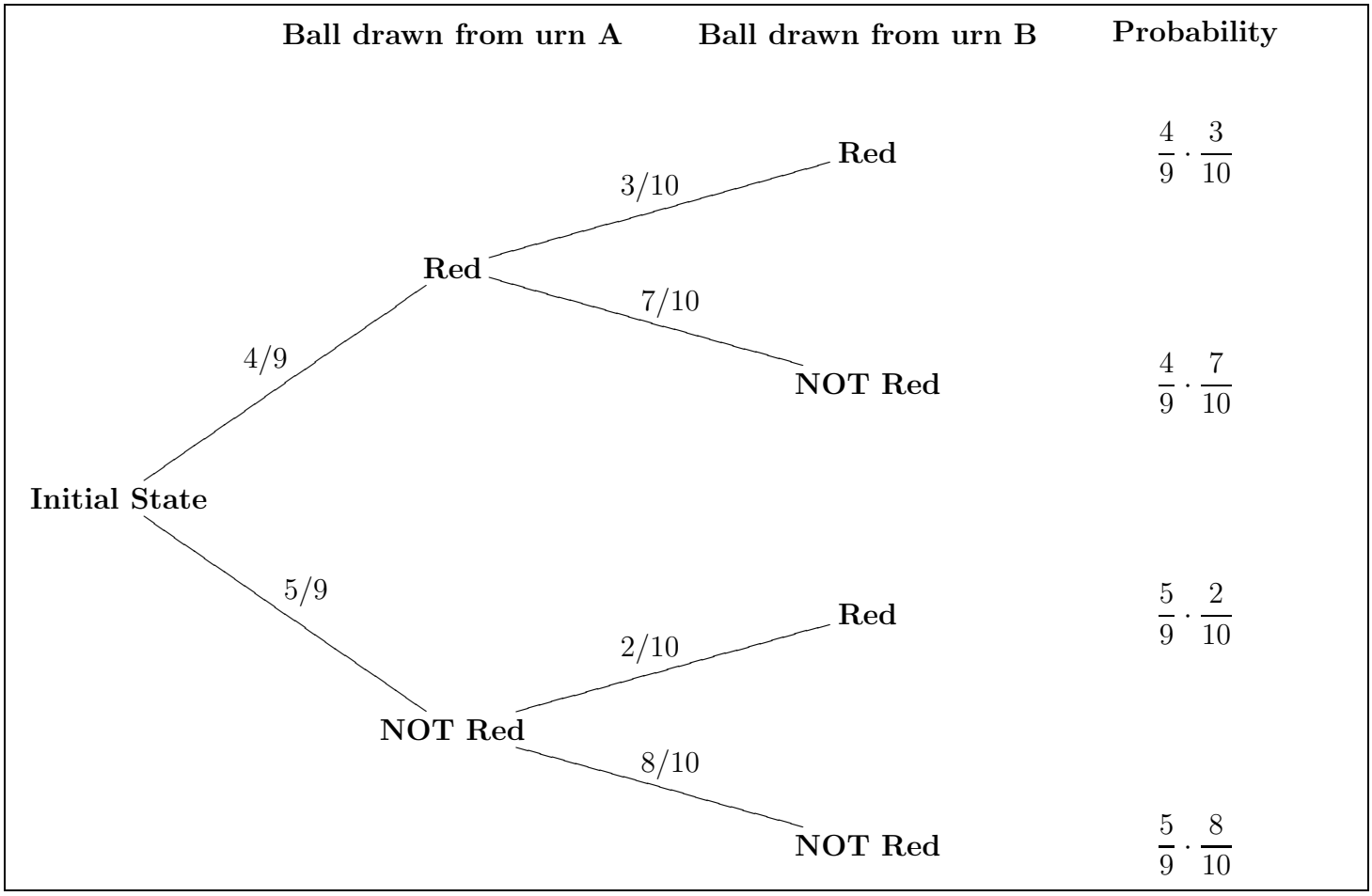
   (a) [4 points] What is the probability that a red ball is drawn from urn B? (show all work)

   Solution: There are total 9 balls in urn A. If the ball drawn from urn A is red, there will be 3 red balls in urn B out of total 10 balls. If the ball drawn from urn

A is not red, there will be 2 red balls in urn B out of total 10 balls. So,

$$
\begin{aligned}
&\mathbf{P}\,[\text{The ball drawn from urn B is red}] \\
&= \mathbf{P}\,[\text{The ball drawn from urn A is red \& The ball drawn from urn B is red}] \\
&\quad + \mathbf{P}\,[\text{The ball drawn from urn A is not red \& The ball drawn from urn B is red}] \\
&= \frac{4}{9} \times \frac{3}{10} + \frac{5}{9} \times \frac{2}{10} \\
&= \frac{12+10}{90} = \frac{\mathbf{22}}{\mathbf{90}} = \mathbf{0.244}.
\end{aligned}
$$

Here is the tree diagram,



(b) [4 points] If a red ball is drawn from urn B, what is the probability that a red ball was drawn from urn A? (show all work.) *Hint: use conditional probability*

Solution:

$$\begin{aligned}
&\mathbf{P}\left[\text{The ball drawn from urn A is red} \,\middle|\, \text{The ball drawn from urn B is red}\right] \\
&= \frac{\mathbf{P}\left[\text{The ball drawn from urn A is red \& The ball drawn from urn B is red}\right]}{\mathbf{P}\left[\text{ The ball drawn from urn B is red }\right]} \\
&= \frac{\frac{4}{9} \times \frac{3}{10}}{\frac{22}{90}} \\
&= \frac{12/90}{22/90} = \frac{\mathbf{12}}{\mathbf{22}} = \mathbf{0.545}.
\end{aligned}$$

3. A committee consists of five Caucasians, two Asians, three African Americans, and two Native Americans. *Hint: can use counting methods rules and* $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ *to answer the following questions*

   (a) [4 points] If a subcommittee of four is chosen at random, what is the probability that all the ethnic groups are represented on the subcommittee? (show all work)

   Solution: There are total 12 members in the committee. So number of ways in which a subcommittee of four can be chosen is $\binom{12}{4} = \frac{12!}{4!8!} = 495$. Number of samples in which there is one member from each ethnic groups is $\binom{5}{1}\binom{2}{1}\binom{3}{1}\binom{2}{1} = 5 \times 2 \times 3 \times 2 = 60$. So the probability that all the ethnic groups are represented on the subcommittee is $\frac{\mathbf{60}}{\mathbf{495}} = \mathbf{0.121}$.

   (b) [4 points] If a subcommittee of five is chosen at random, what is the probability that all the ethnic groups are represented on the subcommittee? (show all work)

   Solution: Number of ways in which a subcommittee of five can be chosen is $\binom{12}{5} = \frac{12!}{5!7!} = 792$. If all the ethnic groups are represented in the subcommittee, there are two members from one group and one member from each of the remaining three groups in the subcommittee. So number of samples in which all the ethnic groups are represented is $\binom{5}{2}\binom{2}{1}\binom{3}{1}\binom{2}{1} + \binom{5}{1}\binom{2}{2}\binom{3}{1}\binom{2}{1} + \binom{5}{1}\binom{2}{1}\binom{3}{2}\binom{2}{1} + \binom{5}{1}\binom{2}{1}\binom{3}{1}\binom{2}{2} = 10 \cdot 2 \cdot 3 \cdot 2 + 5 \cdot 1 \cdot 3 \cdot 2 + 5 \cdot 2 \cdot 3 \cdot 2 + 5 \cdot 2 \cdot 3 \cdot 1 = 240$. So the probability that all the ethnic groups are represented on the subcommittee is $\frac{\mathbf{240}}{\mathbf{792}} = \mathbf{0.303}$.

4. Let A and B be two events that are a subset of a sample space S. Answer each of the following statements with **True or False**.

(a) [1 point] If A and B are independent, then $\mathbf{P}(A \text{ or } B) = \mathbf{P}(A) + \mathbf{P}(B)$.

Solution: FALSE.

[If A and B are independent we have $\mathbf{P}(A \text{ and } B) = \mathbf{P}(A)\,\mathbf{P}(B)$. So $\mathbf{P}(A \text{ or } B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \text{ and } B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A)\,\mathbf{P}(B)$.]

(b) [1 point] If A and B are disjoint, then $\mathbf{P}(A \text{ and } B) = 0$.

Solution: TRUE.

[If A and B are disjoint the event (A and B) is empty.]

(c) [1 point] If A and B are not independent, then $\mathbf{P}(A \text{ and } B) \neq \mathbf{P}(A)\,\mathbf{P}(B|A)$.

Solution: FALSE.

[We always have $\mathbf{P}(A \text{ and } B) = \mathbf{P}(A)\,\mathbf{P}(B|A)$.]