1. (Problem 5.1 from section 7.5 in Durrett)
   Let \( T = \inf\{ t : B_t \notin (-a, a) \} \). Prove that
   \[
   E_0 \exp(-\lambda T) = \frac{1}{\cosh(\sqrt{2}a\lambda)}
   \]
   using the argument that \( \exp(-\theta^2t/2)\cosh(\theta B_t) \) is a martingale.

2. (Problem 5.2 from section 7.5 in Durrett)
   Let \( \tau = \inf\{ t : B_t = a + bt \} \) where \( a > 0 \). Use the martingale \( \exp(\theta B_t - \theta^2 t/2) \) with \( \theta = b + (b^2 + 2\lambda)^{1/2} \) to show that
   \[
   E_0 \exp(-\lambda \tau) = \exp(-a\{b + (b^2 + 2\lambda)^{1/2}\}). \tag{1}
   \]
   Setting \( b = 0 \) gives the formula (4.4) in Durrett. Letting \( \lambda \to 0 \) we get
   \[
   P_0(\tau < \infty) = \exp(-2ab).
   \]

3. (Problem 5.5 from section 7.5 in Durrett)
   Let \( u(t, x) \) be a function that satisfies
   \[
   \frac{\partial u}{\partial t} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} = 0, \quad \left| \frac{\partial^2 u}{\partial x^2}(t, x) \right| \leq C_T \exp(x^2/(t + \varepsilon)) \text{ for } t \leq T. \tag{2}
   \]
   Show that \( u(t, B_t) \) is a martingale by checking that
   \[
   \frac{\partial}{\partial t} p_t(x, y) = \frac{1}{2} \frac{\partial^2}{\partial y^2} p_t(x, y)
   \]
   interchanging \( \partial/\partial t \) and \( \int \), and then integrating by parts twice to show that
   \[
   \frac{\partial}{\partial t} E_x u(t, B_t) = \int \frac{\partial}{\partial t} (p_t(x, y)u(t, y)) dy = 0.
   \]

4. (Problem 11 from chapter 13 in Kallenberg)
   (Paley, Wiener, and Zygmund) Show that Brownian motion is a.s. nowhere Lipschitz continuous, and hence nowhere differentiable.
   **Hint:** If \( B \) is Lipschitz at \( t < 1 \), there exist some \( K, \delta > 0 \) such that \( |B_r - B_s| \leq 2hK \) for all \( r, s \in (t - h, t + h) \) with \( h < \delta \). Apply this to three consecutive \( n \)-dyadic intervals \( (r, s) \) around \( t \).
5. (Problem 17 from chapter 13 in Kallenberg)

Let $\xi_1, \xi_2, \ldots$ are i.i.d. N(0, 1) random variables. Show that

$$\limsup_{n} \frac{\xi_n}{\sqrt{2 \log n}} = 1 \text{ a.s.}$$