Math 561: Theory of Probability I (Spring 2023)

Goals and topics

This is the first half of the basic graduate course in probability theory. The goal of this course is to understand the basic tools and language of modern probability theory. We will start with the basic concepts of probability theory: random variables, distributions, expectations, variances, independence and convergence of random variables. Then we will cover the following topics:

1. basic limit theorems (law of large numbers, central limit theorem and large deviation principle);
2. martingales and their applications;
3. if time allows, we will give a brief introduction to Brownian motion and Stein's method for normal approximation.

Weblinks

Course	go.illinois.edu/math561
Grades	Canvas
Student hours	4–5:50pm Wednesdays + Appointment by email.

Logistics

Instructor	Partha Dey
Office	35 CAB
Contact	By email psdey@illinois.edu with subject line: "Math 561:"
Class	TR 11:00am–12:20pm in room 147 Altgeld Hall.
Grader	Andres Medina Landeros
Textbook	I will post pdf lecture notes for each class. Richard Durrett: Probability: Theory and Examples (Free Online edition v5). We will cover the first four chapters. It is okay to use another edition for studying. Some other relevent books: **P. Billingsley** *Probability and Measure (3rd Edition)*. Chapters 1–30 contain a more careful and detailed treatment of some of the topics of this semester, in particular the measure-theory background. Recommended for students who have not done measure theory.
Prerequisite	The prerequisite for Math 561 is Math 540 – Real Analysis I. We will review measure theory topics as needed. Math 541 is nice to have, but not necessary.
DRES	To obtain disability-related academic adjustments and/or auxiliary aids, students should contact both the instructor and the Disability Resources and Educational Services (DRES) as soon as possible. You can contact DRES at 1207 S. Oak Street, Champaign, (217) 333-1970, or via e-mail at disability@illinois.edu.
Grading Policy	**Homework**: 40% of the course grade. Homework will be assigned weekly on Thursdays on Canvas, to be submitted at the start of next Thursday lecture or earlier in Canvas. Solving a lot of problems is an extremely important part of learning probability. You are encouraged to work together on the homework, but I ask that you write up your own solutions and turn them in separately. **Late homework will not be graded**. If for some reason you've done a homework but can't turn it in online, send it via email before class. Because of this strict policy on late homework, **I will drop your lowest score**. Please talk to the instructor in cases of emergency. **Midterm**: 30% will depend on an in-class midterm exam on (tentatively) **Tuesday, March 28, 2023**. It will be technically comprehensive, but emphasizing recent material up to the most recent graded and returned homework assignment. Exam problems will be similar to homework problems.

Final: 30% will depend on a take home final exam. The **final take home** exam will cover the most important topics of the whole course. It will be assigned on the last day of the class and will be due on (tentatively) **Monday, May 8, 2023**.

Tentative Class Diary

Week		Date	Due	Content
1	Tu	Jan 17		Probability Spaces.
	Th	Jan 19		Measures.
2	Tu	Jan 24		Measures on Real Line.
	Th	Jan 26	HW 1	Random Variables and Distributions.
3	Tu	Jan 31		Expectation.
	Th	Feb 2	HW 2	Properties of Expectation.
4	Tu	Feb 7		Inequalities and Independence.
	Th	Feb 9	HW 3	Fubini's theorem & Pi-Lambda Theorem.
5	Tu	Feb 14		Borel-Cantelli Lemmas.
	Th	Feb 16	HW 4	Strong Law of Large Numbers.
6	Tu	Feb 21		Kolmogorov's maximal theorem.
	Th	Feb 23	HW 5	Applications of SLLN and 3-series theorem.
7	Tu	Feb 28		Convergence in Distribution.
	Th	Mar 2	HW 6	Central Limit Theorems.
8	Tu	Mar 7		Central Limit Theorems contd.
	Th	Mar 9	HW 7	Poisson Convergence.
9	Tu	Mar 21		Helly's Selection Theorem.
	Th	Mar 23	HW 8	Characteristic functions.
10	Tu	Mar 28	Midterm	Sample midterm and solution, Midterm and Solution.
	Th	Mar 30		Conditional expectation.
11	Tu	Apr 4		Regular Conditional Probability.
	Th	Apr 6	HW 9	Martingales, Stopping time.
12	Tu	Apr 11		Wald's Identities, Upcrossing inequalities.
	Th	Apr 13	HW 10	Martingale convergence, Maximal inequalities.
13	Tu	Apr 18		Lp convergence for martingales.
	Th	Apr 20	HW 11	Reverse Martingales.
14	Tu	Apr 25		Optional Stopping, Azuma-Hoeffding ineq.
	Th	Apr 27	HW 12	Concentration inequality.
15	Tu	May 2		Steins method.
16	M	May 8	Final	Take home Final (PDF, TeX)

Emergency information: **PDF** **VIDEO**