Recap: Normal distribution

**Density of** $X \sim N(\mu, \sigma^2)$: 
$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp \left(-\frac{(x-\mu)^2}{2\sigma^2}\right), x \in \mathbb{R}$

with $\mathbb{E}X = \mu, \text{Var}(X) = \sigma^2$.

**Standard Normal Distribution:** $N(0,1)$

**Standardizing:** If $X \sim N(\mu, \sigma^2)$ then $\frac{X-\mu}{\sigma} \sim N(0,1)$.

**CDF of** $N(0,1)$:

$$
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy, \quad -\infty < x < \infty,
$$

and cannot be computed explicitly.

**Compute** $\Phi(x)$: For $x > 0$, $\Phi(x)$ is given in Normal table. For $x < 0$, use

$$
\Phi(x) = 1 - \Phi(-x).
$$
Example 4b

If $X$ is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$, find

(a) $P(2 < X < 5)$;

(b) $P(X > 0)$;

(c) $P(|X - 3| > 6)$.
Normal approximations to binomial

Let $S_n \sim \text{Bin}(n, p)$ be the number of successes in $n$ independent Bernoulli trials. Then, $\mathbb{E}S_n = np$, $\text{Var}(S_n) = np(1 - p)$ and, for large $n$,

$$\frac{S_n - np}{\sqrt{np(1 - p)}} \approx N(0, 1).$$

The approximation is good for $np(1 - p) \geq 10$. Compare to Poisson approximation $p$ is not small here.

Problem 25

Each item produced by a certain manufacturer is, independently, of acceptable quality with probability .95.

Approximate the probability that at most 10 of the next 150 items produced are unacceptable.
Example 4g

The ideal size of a first-year class at a particular college is 150 students. The college, knowing from past experience that, on the average, only 30 percent of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first-year students attend this college.
Exponential random variables

**Definition**

X is an exponential random variable with parameter $\lambda > 0$ if its density is

$$f(x) = \begin{cases} 
\lambda e^{-\lambda x}, & x > 0, \\
0, & x < 0.
\end{cases}$$

Notation: $X \sim \text{Exp}(\lambda)$.

Exponential random variables

If $X \sim \text{Exp}(\lambda)$, then:

$$F(a) =$$

$$\mathbb{E}X =$$

$$\text{Var}(X) =$$
Memoryless Property

**Interest 1:** In practice, an exponential random variable is often the time until some event (earthquake, phone call, etc) occurs.

**Interest 2:** $X \sim \text{Exp}(\lambda)$ has the following *memoryless* property.\(^1\)

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\(^1\)It is the only such random variable – see p. 232 in the textbook.

**Example 5d**

Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 miles.

If a person desires to take a 5000-mile trip, what is the probability that he or she will be able to complete the trip without having to replace the car battery?

What can be said when the distribution is not exponential?
Gamma random variables

**Definition**

$X$ is a Gamma random variable with parameters $\lambda > 0$ and $\alpha > 0$ if its density is

$$f(x) = \begin{cases} 
\frac{\lambda^\alpha e^{-\lambda x} (\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, & x > 0, \\
0, & x < 0,
\end{cases}$$

where $\Gamma(\alpha) = \int_0^\infty e^{-y} y^{\alpha-1} dy$ is the so-called gamma function.

**Notation:** $X \sim \Gamma(\alpha, \lambda)$.

**Interest:** $\Gamma(n, \lambda) = \text{amount of time to wait till } n \text{ events occur (in the Poisson context)}$

Weibull random variables

**Definition**

$X$ is a Weibull random variable with parameters $\nu, \alpha > 0$ and $\beta > 0$ if its c.d.f. is

$$F(x) = \begin{cases} 
0, & x < \nu, \\
1 - e^{-\left(\frac{x - \nu}{\alpha}\right)^\beta}, & x > \nu.
\end{cases}$$

Interest: Weibull is often used to model lifetime of some object, device, individual, etc. Below: $\nu = 0, \beta = c, \alpha = 1$. 

![Graph showing Weibull distributions with different parameters](image-url)
Beta random variables

**Definition**

$X$ is a Beta random variable with parameters $a > 0$ and $b > 0$ if its density is

$$
f(x) = \begin{cases} 
\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}, & 0 < x < 1, \\
0, & \text{otherwise},
\end{cases}
$$

where $B(a, b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx$ is the so-called beta function.

Interest: model random phenomenon with values in some finite interval.
That's all Folks!